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Selected bibliography on Bernard Bolzano's Contributions to Logic and Ontology. Fifth Part: S - Z

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Bernard Bolzano. Annotated Bibliography on His Practical Philosophy

Studies on Bolzano's Logic and Ontology

1. Schnieder, Benjamin. 2007. "Mere Possibilities: a Bolzanian Approach to Non-Actual Objects." *Journal of the History of Philosophy* no. 45:525-550
 "The paper is a detailed reconstruction of Bernard Bolzano's account of merely possible objects, which is a part of his ontology that has been widely ignored in the literature so far. According to Bolzano, there are some objects which are merely possible. While they are neither denizens of space and time nor members of the causal order, they could have been so. Thus, on Bolzano's view there are, for example, merely possible persons, i.e., objects which are neither actual nor persons but which could have been both. In course of the development of Bolzano's views, they are contrasted with the better known theory of his compatriot Alexius Meinong, and it is shown that they have a modern counterpart in the accounts of merely possible objects that were developed by Bernard Linsky and Ed Zalta, and by Timothy Williamson."
 "Here is a brief outline of my paper. The first section is dedicated to the clarification of some basic Bolzanian notions, an understanding of which is needed for what follows. In the second section, I set out to establish that Bolzano in fact had the ontological view I attribute to him. That is, he accepted that there are merely possible objects. The third and final section is concerned with the exposition and reconstruction of Bolzano's account of mere possibilities." (p. 526)

2. ———. 2014. "Bolzano on Causation and Grounding." *Journal of the History of Philosophy* no. 52:309-337
 "This paper is an exploration of Bolzano's views on causation, which have not been thoroughly examined yet. The paper reconstructs Bolzano's position, with a focus on his analysis of the concept of causation, on its ontological presuppositions, and on how he relates causation to his theory of grounding.(1) A comparison with standard positions from the contemporary debate on causation will prove his views to be quite original. Moreover, they are a valuable addition to the more recent debate on metaphysical grounding,(2) in which grounding is sometimes informally described as something like metaphysical causation with the exact connection of the two notions seldom being elaborated. Bolzano's theory explicitly addresses the issue and takes an innovative stance. However, it will also be revealed that his account is beset with problems. But even if his position should ultimately not be tenable, discussing it can deepen our understanding of problems raised in the current debates about causation and grounding and shed new light on them." (p. 309)
 (1) The paper concentrates on general conceptual and metaphysical issues of causation. It will not discuss Bolzano's views on the epistemology of causation, nor his views on detailed matters of fact perhaps better to be treated in physics and its philosophy (such as the question of how causal powers are actually distributed in the world, what kind of basic causal powers there are, etc.).
 (2) See e.g. Rosen, "Metaphysical Dependence"; Schaffer, "What Grounds"; and Fine, "Guide to Ground."
 References
 Fine, Kit. "Guide to Ground." In *Metaphysical Grounding*, edited by F. Correia and B. Schnieder, 37–80. Cambridge: Cambridge University Press, 2012.
 Rosen, Gideon. "Metaphysical Dependence: Grounding and Reduction." In *Modality*, edited by Bob Hale and Avrid Hoffmann, 109–35. Oxford: Oxford University Press, 2010.
 Schaffer, Jonathan. "On What Grounds What." In *Metametaphysics*, edited by David Chalmers et al., 347–383. Oxford: Oxford University Press, 2009.

3. ———. 2022. "A Fundamental Being: Bolzano's Cosmological Argument and Its Leibnizian Roots." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 418-443. New York: Oxford University Press
 "In his ontology, Bolzano uses the notion of grounding to make claims about the dependent and independent existence of entities. In particular, he argues that there must be a fundamental object (in Bolzano's terminology: an *unconditioned* object), whose existence is not grounded in the existence of any other object. In his paper, Benjamin Schnieder reconstructs Bolzano's argument, explains its historical context, and puts the argument under scrutiny." (p. 38)
4. Scholz, Heinrich. 1961. *Concise History of Logic*. New York: Philosophical Library
 On Bolzano see pp. 44-48.
 "Modern logic interprets syllogisms as deduction of judgments from other judgments. Obviously, this interpretation is meaningless so long as we do not know what is meant by deducing one judgment from another. Bolzano did find the relevant interpretation which, it must be owned, *also* does not satisfy us all around but is, nevertheless, epoch-making solely because in pursuing his objective Bolzano turned away from statements and returned to the "forms."(154)
 These "forms" now appear for the first time explicitly in formal logic so that with their aid Bolzano was able to obtain the most interesting interpretations not only for the derivation but also for the rest of the logically basic relations of compatibility, incompatibility, etc.(155) His charming *Philosophische Grammatik*(156) we have already mentioned. A luminous chapter all by itself contains magnificent discussions of earlier treatments of every topic of logic with special reference to Aristotle and Kant.(157) In these discussions there is invaluable material for any *critical history of logic*." (pp. 46-47)
 (154) See above, p. 3-4.
 (155) Cf. especially WL, II, paragraph 154 ff., 198ff.; I, paragraph 95 ff.
 (156) See above, p. 40.
 (157) Cf. the little book of Bolzano's keenly critical pupil which I brought out in 1931 in a new edition together with W. Dubislav and which appeared in the Felix Meiner Verlag in Leipzig. It is F. Prikonsky: *Neuer Anti-Kant oder Prüfung der Kritik der reinen Vernunft nach den in Bolzanos Wissenschaftslehre niedergelegten Begriffen*. Here we also get acquainted with Augustine's anticipation of Bolzano's principles and ideas, a fact hardly commented on to this day.
5. Schubring, Gert. 1993. "Bernard Bolzano -- Not as Unknown to His Contemporaries as Is Commonly Believed?" *Historia Mathematica*:45-53
 Abstract: "An unknown review of Bolzano's three important papers from the years 1816 to 1817 written in 1821 by J. J. I. Hoffmann, a mathematician from Southern Germany, is edited and commented."
 "According to common historiography, Bolzano's pioneer publications, in particular his contributions to a new rigor in analysis in 1816 to 1817, remained almost unknown to the mathematical community. Only one piece of evidence contradicting the general impression that nobody read Bolzano in his own day is frequently quoted: N. H. Abel's remark in one of his *Paris notebooks*. Having read some of Bolzano's publications during the time he spent in Berlin 1825/1826, he noted enthusiastically "Bolzano is a clever man" (1). Abel's appreciation is taken, however, as an isolated instance, and Hermann Hankel is credited with having been the first to bring Bolzano to the general attention of the mathematical community in 1871 (see [Grattan--Guinness 1970, 51-52]).
 (...)
 With regard to this desideratum concerning the history of reception of Bolzano's work in his own time, an essay review of Bolzano's three key papers of 1816/1817 in one of the leading German review journals, the *Jenaische Allgemeine Literatur--Zeitung* (JALZ), is a most welcome find. I came across it when analyzing the JALZ for its numerous mathematical reviews. As a first contribution to the study of

Bolzano's contemporary reception, the essay review is examined in order to explore the reviewer's reading and understanding of Bolzano's work. Moreover, the mathematical education and practice of the reviewer is analyzed, and the role of the transmitting journal is briefly discussed. The essay review itself is also presented, or more precisely, those parts of it that are in the reviewer's own words." (pp. 45-46)

References

Grattan-Guinness, I. 1970. *The development of the foundations of mathematical analysis from Euler to Riemann*. Cambridge, MA: MIT Press.

6. Šebestík, Jan. 1990. "The Archaeology of the Tractatus: Bolzano and Wittgenstein." In *Wittgenstein, eine Neubewertung / Wittgenstein, Towards a Re-Evaluation. Akten des 14. Internationalen Wittgenstein-Symposiums*, edited by Haller, Rudolf and Brandl, Johannes L., 112-118. Wien: Hölder-Pichler-Tempsky
- "In the case of Bolzano, a comparison with Wittgenstein covers not only some specific points, but also the style of their philosophies and the role of logic in the construction of the system. I see three main points of comparison:
1. For Bolzano, formal logic is the central discipline of philosophy: a logical system once set up becomes an instrument for all philosophical analysis.
 2. Bolzano refutes Kant's transcendental argument the function of which is assumed by a logico-semantic theory which is developed in two different ways:
 - a) a theory of meaning or sense based on abstract intensional entities, propositions (*Sätze an sich*) and ideas-in-themselves (*Vorstellungen an sich*). The grammatical forms of ordinary language have to be elucidated and amended in order to comply with canonic forms obtained by the logical analysis of language.
 - b) a theory of reference or denotation, more precisely the logic of classes and the logic of extensional relations between propositions (extensional because defined solely in terms of the truth values of the propositions considered). Particularly important in this respect is the elucidation of fundamental logical notions: validity, contravalidity, logical consequence (deducibility) and its link with probability.
 3. Bolzano's theory of representation (*Vorstellung*) is not properly speaking a picture theory. According to Bolzano, pictures (*Bilder*) are not ideas; they can at most accompany some ideas. No proper functional relationship, no *Abbildung*, is established between propositions and the world. On the one hand, *Sätze an sich* are not *Sachverhalte*, because no *Sachverhalte* correspond to false propositions. On the other hand, the structure of the propositions, which is derived from the structure of the statements of ordinary language, does not correspond exactly to the structure of objects. The system of all true propositions yields a complete description *an sich* of the world and of the properties of things within it, but Bolzano refutes the idea of morphism between the propositions and the world. It is nevertheless on the grounds of Bolzanian theories that the first (Polish-)Austrian picture theory was born. In Twardowski's *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen* (1894), where the author attempts a synthesis between Bolzano's logic and Brentano's descriptive psychology, a functional relationship (*Abbildung*) is established between objects and ideas." (p. 113)
7. ———. 1992. "The Construction of Bolzano's Logical System." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 163-177. Firenze: Leo S. Olschki
- "Several reconstructions of Bolzano's logical system have been proposed until now, some of them at the present workshop. They exploit systematically different aspects of Bolzano's logic and interpret it in terms of different XXth century systems. Such an approach has its own rights, as the full force of Bolzano's logic can be measured only by the standards of our contemporary logic. This is precisely the mark of great authors: each important discovery in their field brings to the light some hitherto unnoticed aspects of their work. That such reinterpretations are possible in the case of Bolzano, that his system can be represented in a quite different conceptual frame and translated into modern symbolic notation simply shows how rich and far reaching are his theories. Another argument favours this approach: a XXth century logician can read Bolzano and other logicians of the past only against the

background of modern theories. It is in this way that the body of scientific knowledge is continuously being transmitted: by adapting and translating incessantly old theories into the present language. Moreover, the very meaning of past theories can often be understood only in the light of our systems. Already Husserl noticed that he would not have been able to grasp the significance of Bolzano's logic if he had not previously studied the most advanced contemporary logical theories - which in his case mainly meant the logic of Schroder!

Nevertheless, this modernizing approach does not yield full justice to Bolzano. Even if some of his doctrines are definitively obsolete, they have their function in the construction of his system. Like his mathematics, his philosophy and his theology, Bolzano's logic was conceived in a specific historical context and its complete understanding requires a close attention to the logical and philosophical theories of his time. This is why a complementary approach seems necessary, namely a historical analysis which would trace the links between his system and the logical doctrines of his contemporaries as well as with great logical theories of the past. My intention is to explain the formation and the structure of his logical system whose core is propositional logic. Bolzano's system of extensional relations between propositions represents one of the decisive innovations in the history of logic. It has no historical antecedents. It is nevertheless connected with logical theories of the late XVIIIth and early XIXth century and my paper tries to elucidate the genesis of Bolzano's system against this historical background. This approach will not only show the originality of Bolzano's achievement in full light, but also give a perhaps unexpected insight into the structure of his logical system.

In my reconstruction, I intend to remain *within* Bolzano's logic, using only conceptual tools which he himself has designed. Therefore, I shall neither attempt to translate his definitions into some XXth century notation, nor confront his logic with our systems. One of the advantages of this approach is to give a presentation of Bolzano's logic which is as simple as possible and has no recourse either to symbolic language (except for elementary set-theoretical notational devices) or to sophisticated semantic framework. Those who have tried to explain Bolzano's logical theories to non-specialists or even to students of modern logic may test the advantage of such an approach." (pp.163-164).

8. ———. 1997. "Bolzano, Exner and the Origins of Analytical Philosophy." *Grazer Philosophische Studien* no. 53:33-59
Abstract: "Analytical philosophy begins with the first mathematical and philosophical works of Bolzano published between 1804 and 1817. There, Bolzano set out a project for the global reform of mathematics by means of the axiomatic method. Having completed the *Wissenschaftslehre*, Bolzano wrote a summary of his logic for the *Grossenlehre*, which he sent to Exner in 1833. The correspondence between Bolzano and Exner covered some of the main subjects treated by analytical philosophy: the status of abstract objects (propositions and objective ideas), intuitions, objectless ideas, the concept of object and many others. While Bolzano argued in favor of abstract entities independent of mind and of language, Exner considered them as abstractions obtained from the subjective judgments and representations. During the XIXth century, Bolzano's philosophy spread over Bohemia and Austria through manuscripts and through the first edition of Zimmermann's textbook of philosophy. The most important Brentanians, Kerry, Twardowski, Meinong and Husserl, discussed his doctrines which may also have influenced Wittgenstein and the Polish school."
9. ———. 2003. "Husserl Reader of Bolzano." In *Husserl's Logical Investigations Reconsidered*, edited by Fiset, Denis. Dordrecht: Kluwer
"The incredible soundness of Husserl's judgment in the matter of logic is unique among his contemporaries - only Frege's insight is on par with it, if not superior. This is due to the lesson of Bolzano whose logic is the truth itself. Husserl adapted his logical system so that it became the logical basis of phenomenology. He adopted Bolzano's main ideas: the extension of logic to the theory of science, the theory of ideal meanings, the distinction between mental act, linguistic expression, meaning

and denoted object, the concept of analyticity. Independently of Bolzano and consonant with later mathematical theories, Husserl developed his formal analytics along two lines, apophantic and formal ontology.

Bolzano, however, had articulated the domain of conceptual truths in the same manner: he constructed his logical system as a theory of meaning and his mathematics as a theory of object in general or *Etwas überhaupt*. Both set theory and mereology have their origin here. By his theory of science, Bolzano gave a new impetus to philosophy and logic. For the first time in modern thought, such questions as the nature of logical objects, the problems of meaning and reference, the relation between logic and language became central issues of philosophy." (p. 80)

10. ———. 2014. "Bolzano's Lehrjahre." In *Mind, Values, and Metaphysics: Philosophical Essays in Honor of Kevin Mulligan. Vol. 1*, edited by Reboul, Anne, 289-293. Dordrecht: Springer
 Abstract: "The paper will discuss some changes in Bolzano's definition of mathematics attested in several quotations from the *Beyträge*, *Wissenschaftslehre* and *Grössenlehre*: is mathematics a theory of forms or a theory of quantities? Several issues that are maintained throughout Bolzano's works will be distinguished from others that were accepted in the *Beyträge* and abandoned in the *Grössenlehre*. Changes will be interpreted as a consequence of the new logical theory of truth introduced in the *Wissenschaftslehre*, but also as a consequence of the overcome of Kant's terminology, and of the radicalization of Bolzano's anti-Kantianism. It will be argued that Bolzano's evolution can be understood as a coherent move, if one compares the criticism expressed in the *Beyträge* on the notion of quantity with a different and larger notion of quantity that Bolzano developed already in 1816. This discussion is based on the discovery that two unknown texts mentioned by Bolzano can be identified with works by von Spaun and Vieth respectively. Bolzano's evolution will be interpreted as a radicalization of the criticism of the Kantian definition of mathematics and as an effect of Bolzano's unaltered interest in the Leibnizian notion of *mathesis universalis*. As a conclusion, it will be argued that Bolzano never abandoned his original idea of considering mathematics as a *scientia universalis*, i.e. as the science of quantities in general, and it will be suggested that the question of ideal elements in mathematics, which has been interpreted as a main reason for the development of a new logical theory, can also be considered as a main reason for developing a different definition of quantity. "
11. Shapiro, Stewart. 2011. "Varieties of Pluralism and Relativism for Logic." In *A Companion to Relativism*, edited by Hales, Steven D., 526-552. Malden: Wiley-Blackwell
 Abstract: "My purpose is to articulate a number of different senses in which one can be a pluralist and/or a relativist concerning logical consequence. I propose, first, that logical consequence is either polysemous or it denotes a cluster concept. In other words, there are a number of different notions that go by that name, often run together, or else there are several aspects of the notions, with varying weights. The different notions, or aspects, of consequence, turn on matters of modality, semantics, effectiveness, justification, rationality, and form. Second, most of the articulations of the pre - theoretic notions(s) of logical consequence make essential use of a boundary between logical and non - logical terminology. This suggests a sort of relativism/pluralism explicitly noted by Bernard Bolzano and Alfred Tarski: logical consequence is relative to the logical/non - logical boundary. An argument may be valid on one collection of logical terms, invalid on another. Third, it is possible that at least some aspects of the notion of logical consequence are vague: there may be borderline cases of valid arguments. If so, we have to turn to what the correct account of vagueness is. On some theories of vagueness, consequence ends up as relative to something, such as a sharpening or a conversational context, and on others, we end up with a kind of pluralism. Finally, there are a number of interesting and important mathematical theories that employ a non - classical logic, and are

- rendered inconsistent if classical logic is imposed. This suggests a fourth kind of relativism/pluralism: relativity to structure." (p. 526)
12. Siebel, Mark. 1997. "Variation, Derivability, and Necessity." *Grazer Philosophische Studien* no. 53:117-137
Abstract: "In Bolzano's view, a proposition is necessarily true iff it is derivable from true propositions that include no intuition (*Anschauung*). This analysis is historically important because it displays close similarities to Quine's and Kripke's ideas. Its systematic significance, however, is reduced by the fact that derivability is defined with recourse to the method of variation, which we are allowed to apply even to propositions containing none of the respective variables. This liberality leads to the result that, according to Bolzano's analysis, every truth is necessarily true. Even by introducing his condition of relevance (shared variables), Bolzano cannot avoid that some propositions come out as necessarily true which are merely contingently true."
 13. ———. 2002. "Bolzano's Concept of Consequence." *The Monist. An International Quarterly Journal of General Philosophical Inquiry* no. 85:581-601
"In the second volume of his *Wissenschaftslehre* (2) from 1837, the Bohemian philosopher, theologian, and mathematician Bernard Bolzano (1781-1848) introduced his concept of consequence, named *derivability* (*Ableitbarkeit*), together with a variety of theorems and further considerations. Derivability is an implication relation between *sentences in themselves* (*Sätze an sich*), which are not meant to be linguistic symbols but the *contents* of declarative sentences as well as of certain mental episodes. When Schmidt utters the sentence 'Schnee ist weiss', and Jones judges that snow is white, the sentence in itself expressed by Schmidt is the same as the one to which Jones agrees in thought. This sentence in itself is an abstract entity: in some sense, it exists; but it is unreal insofar as it lacks a position in space and time, does not stand in causal relationships, and is independent of the existence of thinking beings and languages. (3)" (p. 581)
(*) On the whole, this contribution is a summary of my book *Der Begriff der Ableitbarkeit bei Bolzano* (Siebel 1996).
(2) I refer to it by 'WL' plus number of volume, section, and page. It is partly translated by Rolf George: *Theory of Science*, Oxford 1972; but here translations are mine.
(3) Cf. WL I, § 19, pp. 77f.; § 22, p. 90; § 25, p. 112; § 28, p. 121; WL II, § 122, 4.
 14. ———. 2011. "'It Falls Somewhat Short of Logical Precision.'" Bolzano on Kant's Definition of Analyticity." *Grazer Philosophische Studien* no. 82:91-127
"Kant's famous definition of analyticity states that a judgement is analytic if its subject contains its predicate. Bolzano objects that (i) Kant's definiens permits an interpretation too wide, (ii) the definiens is too narrow, (iii) the definiendum is too limited, and (iv) the definition does not capture the proper essence of analyticity. Objections (i), (iii) and (iv) can be countered. Objection (ii) remains because, among other things, the Kantian definition has an eye only for an analysis of the subject within a judgement."
 15. ———. 2019. "Bolzano's Theory of Judgment." In *The Act and Object of Judgment: Historical and Philosophical Perspectives*, edited by Ball, Brian and Schuringa, Christoph, 110-128. New York: Routledge
"Section 2 presents one of the many places where Bolzano anticipates Frege's anti-psychologistic notion of a third realm, which complements the inner realm of mental appearances and the outer realm of perceivable objects. In particular, Bolzano strictly distinguishes between judgments as mental acts and the contents of such acts. In section 3, it is shown how he tries to draw the line between judgments and acts of merely entertaining a thought. Section 4 focuses on the formation of judgments. Of prime importance is the distinction between mediated and unmediated judgments because it is intimately connected with epistemic issues. Section 5 deals with intrinsic qualities of judgments, such as vividness, degree of confidence, clarity vs obscurity, and distinctness vs confusedness.(1)

The notion of judgment occupies centre stage in Bolzano's analyses of epistemic concepts. It is not only crucial to his explication of belief (*Meinung*) as a disposition to judge but also to his explications of cognition (*Erkenntnis*) as true judgment and conviction (*Überzeugung*) and knowledge (*Wissen*) as attitudes towards judgments. In the interest of brevity, I will not go into this conceptual enterprise. Instead, it will be pointed out that Bolzano's theory of judgment includes ingredients one would hardly expect when being told that he anticipated Frege's antipsychologistic views." (pp. 110-111)

(1) Some of the following considerations may also be found in Siebel (1999) and Siebel (2004).

References

Siebel, M. (1999), "Bolzanos Erkenntnistheorie", in E. Morscher (ed.), *Bernard Bolzanos geistiges Erbe für das 21. Jahrhundert*. Sankt Augustin: Academia.

Siebel, M. (2004), "Bolzanos Urteilslehre", *Archiv für Geschichte der Philosophie* 86: 56–87.

16. Simons, Peter. 1987. "Bolzano, Tarski, and the Limits of Logic." *Philosophia Naturalis*:378-405
Reprinted in: Peter Simons, *Philosophy and logic in Central Europe from Bolzano to Tarski. Selected Essays*, Dordrecht, Kluwer 1992, pp. 13-40.
Abstract: "Both Bolzano and Tarski were unsure what counts as logic. This means that Bolzano's concept of logical analyticity, like Tarski's of logical consequence, is not completely determinate.
In a posthumously published paper, Tarski offers a proposal for demarcating the logical objects in a type-hierarchy, based on the idea of invariance under arbitrary permutations of the domain of individuals. In this paper I comment on and extend Tarski's proposal and show how to combine it with Bolzano's procedure of variation among concepts, to obtain a definition of logical constants in a logically significant fragment of a purported Bolzanian realm of meanings in themselves. I conclude with doubts about the propriety and utility of such a realm."
17. ———. 1997. "Bolzano on Collections." *Grazer Philosophische Studien* no. 53:87-108
Abstract: "Bolzano's theory of Collections (*Inbegriffe*) has usually been taken as a rudimentary set theory. More recently, Frank Krickel has claimed it is a mereology. [*] I find both interpretations wanting. Bolzano's theory is, as I show, extremely broad in scope; it is in fact a general theory of collective entities, including the concrete wholes of mereology, classes-as-many, and many empirical collections. By extending Bolzano's ideas to embrace the three factors of kind, components and mode of combination, one may develop a coherent general account of collections. But it is most difficult to take Bolzano's view to fit modern set theory. So while Krickel's positive thesis is rejected, his negative thesis is confirmed."
F. Krickel, *Teil und Inbegriff. Bernard Bolzanos Mereologie*, 1995.
18. ———. 1999. "Bolzano, Brentano and Meinong: Three Austrian Realists." In *German Philosophy Since Kant*, edited by O'Hear, Anthony, 109-136. Cambridge: Cambridge University Press
"Bolzano's work will in due course be wholly accessible in print and should present relatively few problems of interpretation. I foresee a steadily growing reputation, but whether he comes to his just recognition will depend on attracting sufficiently many interested and talented commentators. The most promising centre of Bolzano studies is currently Hamburg, where a number of young enthusiasts have gathered around Wolfgang Kühne.
Of the three philosophers I have mentioned, Bolzano is without doubt the most considerable. Meinong's theories are in the end unacceptably extreme and Brentano's work is often unclear in its implications, though both say things which are of much value to present-day discussions. On the other hand, whether one agrees with his semantic Platonism or not, Bolzano's views are up to the highest standards of contemporary discussion and in their clarity above much of it. His

- correspondence with Ferdinand Exner has been called the first text of modern analytical philosophy. Most work has to date concentrated on his logic and semantics, but his ethics, political philosophy, philosophy of religion and philosophy of mathematics all deserve greater exposure. The Complete Edition will serve as a definitive textual basis, but it is very expensive, and we badly need cheap study texts in English and German to complement it, and a good introduction to Bolzano in English. We also need to revise our histories of nineteenth-century philosophy to take adequate account of its greatest representative." (p. 126)
19. ———. 2006. "Austrian Philosophers on Truth." In *The Austrian Contribution to Analytic Philosophy*, edited by Textor, Mark, 159-183. New York: Routledge
 "In this chapter, I shall consider what the principal Austrian philosophers from Bolzano to Popper have had to say on the subject of truth. Since I shall cover a fair number of philosophers and theories, my considerations will be mainly confined to two linked questions:
 What – according to the philosopher in question – is the nature of truth?
 What ontology is required to explicate truth according to their account?
 Further questions concerned with our access to and knowledge of the truth will only be considered as necessary, since they lead into a tangle of issues for which I shall not have the space here. Neither shall I justify my selection of this or that philosopher as ‘Austrian’, but simply press on." (p. 159)
 20. ———. 2011. "Bolzano's Logic." 1-19
 Available on the website academia.edu
 Original translated by Giorgio Volpe and published in Italian as “Bolzano e la logica” in S. Besoli, L. Guidetti and V. Raspa, eds., *Bernard Bolzano e la tradizione filosofica*. Macerata: Quodlibet. = *Discipline filosofiche XXI*, 2, 2011, 321–342.
 Abstract: "Bolzano's Wissenschaftslehre (1837) is one of the two most important works in logic between Leibniz and Frege. In it, Bolzano revolutionised logic by placing it for the first time on a firm semantic footing, employing the concepts of objective, abstract propositions and ideas. The chief instrument in his account of logic is the variation of ideas, which enabled him to define a wide range of logical concepts, and further allowed him to merge deductive logic with a logical conception of probability. This article summarizes the main points of Bolzano's logic and indicates ways in which they relate to post-Fregean logic."
 21. ———. 2015. "Bolzano's Monadology." *British Journal for the History of Philosophy* no. 23:1074-1084
 Abstract: "Bernard Bolzano (1781–1848), known in his lifetime as ‘the Bohemian Leibniz’, is best known as a logician and mathematician, but he also developed a monadology in which the monads, which he called ‘atoms’, have spatial location and physical properties. This essay summarizes and assesses his monadology."
 22. Smart, Harold R. 1944. "Bolzano's Logic." *The Philosophical Review* no. 53:513-533
 "Contemporary advocates of Husserl's phenomenological approach to the problems of philosophy tend, consciously or unconsciously, to convey the impression that there is only slight connection between Bernard Bolzano's logical theories and those of their Master. Unfortunately their attitude on this matter encourages the common belief that Bolzano may be safely ignored by students of logic-that his work in this field is of little consequence at the present time. Yet in Husserl's own estimation Bolzano was one of the greatest logicians of all times, and historians of philosophy have called him a "Leibniz auf böhmischen Boden".
 He was at all events one of the staunchest opponents of the metaphysical logicians following Kant, as well as of Kant himself.
 His *Wissenschaftslehre* (1837), a compendious work in four volumes totalling nearly 2500 pages, draws much of its inspiration from Augustinian and Leibnizian sources, and in turn has served as a basis for certain theories of Brentano, Husserl, Meinong, and others. Like Leibniz he zealously occupied himself with both

- mathematics and philosophy from early youth, and again like Leibniz he is rightly famous for his distinguished work in both fields. Indeed his *Paradoxien der Unendlichen* (posth. 1850) is said to have started the great Cantor on his researches in the realm of the mathematical infinite. And he is another of the few thinkers whose chief philosophical writings are in the field of logic. For the rest, his writings are shot through with references to his predecessors, both ancient and modern, and with critical remarks on their doctrines." (p. 513)
23. Stang, Nicholas F. 2013. "A Kantian Reply to Bolzano's Critique of Kant's Analytic-Synthetic Distinction." *Grazer Philosophische Studien* no. 85:33-61
Summary: "One of Bolzano's objections to Kant's way of drawing the analytic-synthetic distinction is that it only applies to judgments within a narrow range of syntactic forms, namely, universal affirmative judgments. According to Bolzano, Kant cannot account for judgments of other syntactic forms that, intuitively, are analytic. A recent paper by Ian Proops also attributes to Kant the view that analytic judgments beyond a limited range of syntactic forms are impossible. I argue that, correctly understood, Kant's conception of analyticity allows for analytic judgments of a wider range of syntactic forms."
24. ———. 2014. "Kant, Bolzano, and the Formality of Logic." In *New Anti-Kant*, edited by Lapointe, Sandra and Tolley, Clinton, 192-234. London: Palgrave Macmillan
"In §12 of his 1837 *magnum opus*, the *Wissenschaftslehre*, Bolzano remarks that "In the new logic textbooks one reads almost constantly that 'in logic one must consider not the material of thought but the mere form of thought, for which reason logic deserves the title of a purely formal science'" (*WL* §12, 46).(1) The sentence Bolzano quotes is his own summary of others' philosophical views; he goes on to cite Jakob, Hoffbauer, Metz, and Krug as examples of thinkers who held that logic abstracts from the matter of thought and considers only its form. Although Bolzano does not mention Kant by name here, Kant does of course hold that "pure general logic", what Bolzano would consider logic in the traditional sense (the theory of propositions, representations, inferences, etc.), is formal.
(...)
In recent work, both John MacFarlane and Sandra Lapointe have argued that this 'formality thesis' is original to Kant; according to them, no one in the pre-Kantian, Leibnizian logical tradition held that logic is about the form of thinking.(3) As MacFarlane points out, the claim that logic is formal is now so widespread that it is often simply asserted without argument. So in criticizing the formality thesis in these post-Kantian figures (whom Lapointe aptly dubs 'Kantian logicians') Bolzano is really targeting one of Kant's most influential ideas in the philosophy of logic." (pp. 192-193)
(1) References to the *Wissenschaftslehre* (*WL*) are to Bolzano (1837); it is cited by section number and page.
(3) MacFarlane (2002) and Lapointe (2012).
References
Lapointe, S. (2012). 'Is Logic Formal? Bolzano, Kant and the Kantian Logicians', *Grazer Philosophische Studien*, 85, 11–32.
MacFarlane, J. (2002). 'Frege, Kant, and the Logic in Logicism', *The Philosophical Review*, 111, 25–65.
25. Stelzner, Werner. 2002. "Compatibility and Relevance: Bolzano and Orlov." *Logic and Logical Philosophy* no. 10:137-171
"Ivan Orlov (1886 - not later 1936) is the author of "The Logic of Compatibility of Propositions", *Matematicheskii Sbornik* 35, 1928, pp. 263-86 (in Russian), "the first precisely elaborated modern system of relevance logic" (p. 137)
"In Bernard Bolzano Orlov had a great predecessor in the attempt of deriving the concept of logical consequence, and indeed of relevant consequence, from the concept of compatibility of sentences. It is appropriate, therefore, to turn to Bolzano

- in order to check out parallels and divergences in the treatment and role of the compatibility of sentences in Bolzano's and Orlov's logical projects." (p. 142)
26. Sundholm, Göran. 1994. "Ontologic versus Epistemologic: some Strands in the Development of Logic 1837-1957." In *Logic and Philosophy of Science in Uppsala*, edited by Prawitz, Dag and Westerståhl, Dag, 373-384. Dordrecht: Kluwer
- "Inferences, that is, acts of passage in which a certain judgement, the conclusion of the inference, is drawn on the basis of certain already made judgements, the premisses of the inference, have yielded their central place at the hard core of logic to relations of logical consequence between propositions that serve as contents of the judgements involved, or even more commonly, between well-formed formulae, that is, between meta-mathematical objects of an uninterpreted formal language. In the present paper I intend to review some of the steps in the process whereby this came about, as well as mention a couple of philosophical corollaries.
- Quine, in 1952, held that 'logic is an old subject and since 1879 it has been a great one'.(1) No one reasonably informed concerning the development of logic could possibly object to the first part of this statement, but I want to take mild exception to the second: logic was great also prior to the appearance of Frege's *Begriffsschrift*. (2) From the perspective I am concerned to develop here, 1837 is as important a year as 1879. In that year Bernhard Bolzano's *Wissenschaftslehre* made its appearance in four mighty tomes.(3)" (pp. 373-374)
- (1) *Methods of Logic*, Holt and Co., N.Y. 1950, p. vii.
 (2) Louis Nebert, Halle, Jena 1879.
 (3) J. von Seidel, Sulzbach.
27. ———. 1998. "MacColl on Judgement and Inference." *Nordic Journal of Philosophical Logic* no. 3:119-132
- "The theme of our conference is that of Hugh MacColl and the logical tradition. From any point of view, surely, judgement and inference are (possibly *the*) central components of the logical tradition. However, they do not occur as such in MacColl's Symbolical reasoning(s).
 (...).
 Accordingly, I begin with a rational reconstruction of what I see as the pivotal moment in the 19th century logical tradition, namely Bolzano's introduction of a novel form of judgement, which will be used to take the measure of the early MacColl with respect to judgement and inference." (p. 119)
- (...)
 !Why does this Bohemian priest [Bolzano] deserve pride of place over and above such luminaries as Boole, Peirce and Frege? For more than two thousand years, logic has been concerned with how to effect valid acts of inference from judgements known to other judgements that become known through the inference in question. Basically, these judgements take the subject/copula/predicate form [S is P]. Bolzano now has the courage to break with this traditional pattern and uses instead the unary form
- (1) A is true;
 where A is a *Satz an sich*, or a *Gedanke*, in the later alternative terminology of Frege. The latter term was translated into English as *proposition* by Moore and Russell, with an unusually confusing ambiguity as a result: prior to 1900 a "proposition" stood for a judgement (made), whereas later it came to stand for the propositional content of such a judgement." (p. 120)
28. ———. 1999. "When, and Why, did Frege read Bolzano?" In *Logica Yearbook 1999*, 164-174
- "Michael Dummett wrote:
 The only nineteenth-century philosopher of whom it would be reasonable to guess, just from the content of his writings and those of Frege, that he had influenced Frege, is Bernhard Bolzano, who died in the year Frege was born; but there is no evidence whatever that Frege ever read Bolzano(1)

Subsequently he was taken to task by Wolfgang Künne for having made the 'grave mistake' of misspelling 'Bernard', the first name of Bolzano.

However, in my opinion, this is not the only mistake in the quote from Dummett. In the present note I wish to dispute that 'there is no evidence whatever that Frege ever read Bolzano'. On the contrary, by combining two well-known sets of facts, I shall argue, one obtains strong evidence that Frege did read Bolzano late 1905 or early 1906." (p. 164, a note omitted)

(...)

"On the strength of internal evidence I have argued that Frege did read Bolzano.

Was it in fact possible for him to do so? It certainly was, as Dr. Uwe Dathe, of the Philosophical Institute at Jena University, has been kind enough to check.(26) The University Library at Jena owns a set of Bolzano's collected works from 1882. The acquisition is not dated, but from the library stamp and binding it is clear that the set must have been obtained shortly after its appearance.

Unfortunately, the library ledgers for the years 1821-1899, which have miraculously been retained, are in too bad a state to allow for any conclusion whether Frege actually borrowed the work during that period.(27)

Finally, if, as I aver, Frege did read Bolzano, why does he not simply say so? The answer here surely lies in his character: throughout his career Frege *never* acknowledges, but always disagrees.(28) His spirit seems to have been essentially adversarial. He is the typical *Gegner* who only attacks, but who cannot be bothered to agree." (p. 172)

(1) *Frege and Other Philosophers*, Clarendon Press, Oxford, 1991, p. VII.

(2) 'Propositions in Bolzano and in Frege', in: *Grazer Philosophische Studien* (Bolzano and Analytic Philosophy; edited by Wolfgang KUNne, Mark Siebel and Mark Textor) 53 (1997), pp. 202-240, at p. 203.

(26) Private letter, November 26, 1998.

(27) Of course, if I am right, a later loan, in 1905 or 1906, outside the period of the ledgers, would be more likely.

29. ———. 2002. "A Century of Inference: 1837-1936." In *In the Scope of Logic, Methodology and Philosophy of Science. Vol. II*, edited by Gardenfors, Peter, Wolenski, Jan and Kijania-Placek, Katarzyna, 565-580. Dordrecht: Kluwer
- "The first serious breach in the traditional logical fortress was broached by one thoroughly steeped in the Scholastic patrimony, namely Bernard Bolzano, in another *Wissenschaftslehre* from 1837. This, however, is no puny pamphlet, but a monumental four-volume tome.(5) Like all good ideas the basic idea behind Bolzano's magisterial change is essentially simple: Bolzano revolutionizes logical theory by "objectivizing" the middle column of the traditional diagram. This objectivization consists in severing the left - and right-hand links to mind and language, thereby obtaining objective "Platonist" logical notions, for which Bolzano ironically adopts the Kantian 'an sich' idiom.
- Thus, the (mental) terms become objective "ideas-in-themselves" (*Vorstellungen an sich*). (6) The judgements made, that is, the mental propositions, become propositions-in-themselves (*Sätze an sich*), that is, propositions in the modern, post-Russellian sense.(7) Finally, the mental inferences are replaced by *Ableitbarkeiten*, that is, relations of (logical) consequence between propositions-in-themselves. The resulting change with respect to the form of judgement is particularly interesting. In place of the traditional bipartite Subject/copula/Predicate form Bolzano uses the unary form
- C* is true,
- where *C* is a *Satz an sich*, that is, a proposition that serves as content of the judgement in question (WL §34). The form of the proposition *C*, on the other hand, stays close to the traditional [*S* is *P*]. Bolzano uses [*A* has *b*], where *A* and *b* are *Vorstellungen an sich*, that is, (what corresponds to) objectivizations of the mental products of simple apprehensions, as canonical form for the objective propositions. Thus, he converts the traditional form of judgement into a form of content:
- The proposition that the rose has redness is true

instead of

The rose is red." (pp. 567-568. a note omitted)

(7) Russell ([*Principles of Mathematics*] 1903, Appendix A) might be responsible for sanctioning the unfortunate use of the term Russell (1903, Appendix A) might be responsible for sanctioning the unfortunate use of the term *proposition* for the Fregean *Gedanken*.

30. ———. 2009. "A Century of Judgement and Inference: 1837-1936. Some Strands in the Development of Logic." In *The Development of Modern Logic*, edited by Haaparanta, Leila, 263-318. New York: Oxford University Press

§ 3. Revolution: Bolzano's *Annus Mirabilis*, pp. 269-273.

"My office in the present chapter is to tell how, within a century, the notions of judgment and inference were driven out of logical theory and replaced by propositions and (logical) consequence. Systematic considerations guide the treatment. My history is unashamedly Whiggish: A current position will be shown as the outcome, or even culmination, of a historical development. No apology is offered, nor, in my opinion, is one needed." (p. 263)

(...)

"Bolzano's revolution with respect to the traditional picture is threefold.

First, the middle ("product") column of the traditional schema is objectified.

The mental links are severed, and thus, in particular, the traditional notions mental term (concept, idea) and mental proposition (judgment) are turned into their ideal,

or Platonist, counterparts idea-in-itself (*Vorstellung an sich*) and proposition-in-itself (*Satz an sich*).⁽²³⁾ Second, the pivotal middle square of the diagram is altered:

The judgment made no longer takes the traditional (*S is P*) form. Logic is no longer term logic. Instead Bolzano uses the propositional, unary form of judgment that was canvassed above, with his *Sätze an sich* taking the role of judgable contents:

The *Satz an sich* *S* is true.⁽²⁴⁾

Third, Bolzano bases his logical theory, not on inference (from judgments known to judgment made), but on (logical) consequence between propositions.⁽²⁵⁾

Judgment is dethroned and its content now holds pride of place in logical theory." (pp. 269-270)

(23) The English rendering of Bolzano's *Satz an sich* is a matter of some delicacy.

The modern, Moore-Russell notion of proposition, being an English counterpart of the Fregean Thought (German *Gedanke*), really is an *an sich* notion, and, for our purposes, essentially the same as Bolzano's *Satz an sich*. Thus, proposition-in-itself is pleonastic: The in-itself component is already included in the proposition.

Furthermore, the mental propositions and their linguistic signs, that is, written or spoken propositions, as explained, carry assertoric force, whereas Bolzano's *Sätze an sich* manifestly do not, serving, as they do, in the role of judgmental content. (...)

(24) WL, §34.

(25) Occasionally I shall permit myself to drop the "in-itself" idiom in the interest of perspicuity and readability and speak just of "propositions."

31. Świątorzecka, Kordula. 2017. "Bolzano's Argument for the Existence of Substances: a Formalization with Two Types of Predication." *Acta Analytica* no. 32:411-426

Abstract: "The topic of our analysis is the argument for the existence of substances given by Bernard Bolzano in *Athanasia* (1827), where he essentially employs two ontological categories: substance and adherence. Bolzano considers the real and conditioned *Inbegriff* of all adherences, which are *wirklich* and *nicht selbst bestehen*.

He claims that the formed collection is dependent on something external and nonadherential, which therefore is a substance. Bolzano's argumentation turns out to be structurally similar to his argument for the existence of God from *Lehrbuch der Religionswissenschaft* (1834), but in each of these reasonings, we find different plausible interpretations of the key concept "Inbegriff". The latter argumentation refers to the mereological totality of existentially conditioned objects. We propose the explication of the Bolzanian *Inbegriff* of all adherences using two types of

predication: we consider its extension as composed of certain intensional counterparts of adherences.

In our approach, we use a fragment of the theory of abstract objects formulated by E. Zalta (1983), describing two different relations between individuals and properties: extensional exemplification and intensional encoding. We put our reconstruction in a wider context of Bolzano's ontology, formulating the needed axioms with two primitive predicates of second order ... is an adherence, ... is conditioned by something real as well as the conditionally introduced first order predicate constant *In* for *Inbegriff* of all adherential ideas. Finally, we sketch a model for our theory."

References

Zalta, E. (1983). *Abstract object: an introduction to axiomatic metaphysics*, D. Dordrecht: Reidel.

32. ———. 2019. "Two Formal Interpretations of Bolzano's Theory of Substances and Adherences." *Axiomathes* no. 29:265-284
 Abstract: "Our research concerns a formal representation of Bolzano's original concepts of *Substanz* and *Adhärenz*. The formalized intensional theory enables to articulate a question about the consistency of a part of Bolzano's metaphysics and to suggest an answer to it in terms of contemporary model theory. The formalism is built as an extension of Zalta's theory of abstract objects, describing two types of predication, viz. attribution and representation. Bolzano was aware about this distinction.
 We focus on the consistency of this formalism and the description of its semantics. Firstly, we explore the possibility to reconstruct a Russellian antinomy based on the concept of the Bolzano's *Inbegriff* of all adherences. (Bolzano's theory of ideas is often suspected of antinomical consequences.) Our aim is to show limitations of his theory that prevent a contradiction when the *Inbegriff* consists of non-selfreferential adherences. Next, we discuss two competing semantics for the proposed theory: Scott's and Aczel's semantics. The first one yields a problematic result, that there are no models for the considered theory, containing a non-empty collection of all adherences. This is due to the fact that Scott's structures verify the formula on reloading abstracts in extensional contexts. We show that Aczel's semantics does not contain this difficulty. There are described Aczel's models with a non-empty set of all adherences. The self-referentiality of such a collection becomes irrelevant here. Finally, we show that there are Aczel's structures verifying the formula on reloading abstracts and we exclude them from the class of models intended for our theory."
 References
 Zalta, E. (1983). *Abstract object: an introduction to axiomatic metaphysics*, D. Dordrecht: Reidel
33. Tarski, Adam. 2002. "On the Concept of Following Logically." *History and Philosophy of Logic* no. 23:155-196
 Translated from the Polish and German by Magda Stroinka and David Hitchcock.
 "We provide for the first time an exact translation into English of the Polish version of Alfred Tarski's classic 1936 paper, whose title we translate as 'On the concept of following logically'.
 We also provide in footnotes an exact translation of all respects in which the German version, used as the basis of the previously published and rather inexact English translation, differs from the Polish. Although the two versions are basically identical, to an extent that is even uncanny, we note more than 400 differences. Several dozen of these are substantive differences due to revisions by Tarski to the Polish version which he did not incorporate in the German version.
 With respect to these revisions the Polish version should be regarded as more authoritative than the German. Hence scholars limited to an English translation should use ours." (p. 1)
 "After the original of this paper had appeared in print, H. Scholz in his article 'Die Wissenschaftslehre Bolzanos, Eine Jahrhundert-Betrachtung', *Abhandlungen der Fries'schen Schule*, new series, vol. 6, pp. 399-472 (see in particular p. 472,

- footnote 58) pointed out a far-reaching analogy between this definition of consequence and the one suggested by B. Bolzano about a hundred years earlier." [Note added by Tarski in English in Tarski (1956, 1983).] (p. 67).
34. Tatzel, Armin. 2002. "Bolzano's Theory of Ground and Consequence." *Notre Dame Journal of Formal Logic* no. 43:1-25
Abstract: "The aim of the paper is to present and evaluate Bolzano's theory of grounding, that is, his theory of the concept expressed and the relation brought into play by 'because'. In the first part of the paper (Sections 1-4) the concept of grounding is distinguished from and related to three other concepts: the concept of an epistemic reason}, the concept of causality, and the concept of deducibility (i.e., logical consequence). In its second part (Sections 5-7) Bolzano's positive account of grounding is reconstructed in axiomatic form and critically discussed."
 35. Textor, Mark. 1997. "Bolzano's Sententialism." *Grazer Philosophische Studien* no. 53:181-202
Abstract: "Bolzano holds that every sentence can be paraphrased into a sentence of the form "A has b". Bolzano's arguments for this claim are reconstructed and discussed. Since they crucially rely on Bolzano's notion of paraphrase, this notion is investigated in detail. Bolzano has usually been taken to require that in a correct paraphrase the sentence to be paraphrased and the paraphrasing sentence express the same proposition. In view of Bolzano's texts and systematical considerations this interpretation is rejected: Bolzano only holds that the sentence to be paraphrased and the paraphrasing sentence must be equipollent ("*gleichgeltend*"). It is shown that even this modest view of paraphrase does not help Bolzano in sustaining his claim that all sentences have the form "A has b"."
 36. ———. 2001. "Logically Analytic Propositions *A Posteriori*?" *History of Philosophy Quarterly* no. 18:91-113
"In this paper I will be concerned with Bolzano's explication of logical analyticity or I-analyticity for short. Nowadays Bolzano is often seen as a forerunner of the so-called substitutionalist account (Etchemendy) of I-analyticity for sentences, the property that distinguishes logical truths (falsehoods) from "ordinary" truths (falsehoods). I will argue that Bolzano's explication does not correspond closely to the modern account. My reason for this heterodox view is not that Bolzano tries to define what makes a *proposition*, roughly, the meaning of a sentence, I-analytic. The problem I am interested in will also arise for an account of I-analyticity for sentences that follows Bolzano's lead. My reason is an epistemological one: Bolzano's account does not allow him to say that I-analytic propositions can be known *a priori*. But according to most philosophers' understanding of I-analyticity this epistemological feature is central to the notion of logical truth. Hence, Bolzano's account does not capture an important feature of the concept of a logical truth or the broader concept of an I-analytic proposition." (p. 91)
 37. ———. 2003. "'Caius-at-Noon" or Bolzano on Tense and Persistence." *History of Philosophy Quarterly* no. 20:81-102
Translated in French as: "Bolzano sur le temps et la persistence", *Philosophiques*, 30, 2003, pp. 105-125.
"Bolzano's fame among contemporary analytic philosophers is mainly due to his achievements in the philosophy of logic.
(...)
What is less well known is that Bolzano also uses his theory of propositions to define a variety of epistemological and metaphysical notions. Among the metaphysical notions so defined is the notion of time. Crucial for his definition of time is Bolzano's thesis that
a tensed natural language sentence attributing a substantial property to an actual thing expresses only a complete proposition if it contains an expression like "in (at) t" as part of its *subject-term*.
Bolzano consequently rejects the Aristotelian idea that tense attaches to predicables.
(1) Bolzano's proposal is of interest for contemporary philosophers, because it bears

a striking resemblance to contemporary theories in which expressions like "Caius at noon" refer to temporal parts. This paper is primarily concerned with a reconstruction and evaluation of the part of Bolzano's doctrine of propositions that is the basis of his definition of time. The definition itself will be a topic for another occasion. First things first. The following sketch of Bolzano's definitional strategy and its rationale shall introduce the reader to Bolzano's general project, which connects tense and time." (pp. 81-82)

(1) Aristotle puts his view forward in *De Interpretatione* 16^b6 and 16^b8.

For recent defenses see P. T. Geach, *Reference and Generality* (Ithaca, N.Y.: Cornell University Press, 1962), §27, and D. Wiggins "Substance," in *Philosophy*, ed. A. C. Grayling (Oxford: Oxford University Press, 1995), p. 232.

38. ———. 2013. "Bolzano on the Source of Necessity: A Reply to Rusnock." *British Journal for the History of Philosophy* no. 21:381-392
 Abstract: "According to Bolzano, an object has necessary being if, and only if, there is a conceptual truth that ascribes being to it. I (Textor, 1996, chapter 5) proposed that the notion of conceptual truth bears the explanatory weight in Bolzano's theory of necessity because, ultimately, the truth of such a proposition depends only on the nature of the concepts it contains. Rusnock (2012) argues against this interpretation and proposes, in turn, that for Bolzano necessity and contingency are tied to free choice. In this article I will provide conceptual and historical background for Bolzano's view of necessity and use it to motivate my interpretation as well as to rebut Rusnock's criticism."
 References
 Rusnock, P. 'Remarks on Bolzano's Conception of Necessary Truth', *British Journal for the History of Philosophy*, 20, 817-837, (2012).
 Textor, M. *Bolzano's Propositionalism* (Berlin/New York: De Gruyter, 1996).
39. ———. 2013. "Bolzano's Anti-Kantianism: From a Priori Cognitions to Conceptual Truths." In *The Oxford Handbook of The History of Analytic Philosophy*, edited by Beaney, Michael, 227-250. New York: Oxford University Press
 Abstract: "Bernard Bolzano was born in 1781, the year of the publication of the first edition of Kant's *Critique of Pure Reason*; he died in 1848, the year of Gottlob Frege's birth. These dates are symbolic. Bolzano's work is a link between Kant's philosophy and early analytic philosophy of which Frege is a key exponent. In this chapter I will discuss how Bolzano's criticism of Kant shapes Bolzano's theory of propositions. In connection with this I will outline how Bolzano discovered the method of variation and give an overview of his results in employing this method."
40. ———. 2013. "Bolzano on Conceptual and Intuitive Truth: the Point and Purpose of the Distinction." *Canadian Journal of Philosophy* no. 43:13-36
 Abstract: "Bolzano incorporated Kant's distinction between intuitions and concepts into the doctrine of propositions by distinguishing between conceptual (*Begriffssätze an sich*) and intuitive propositions (*Anschaungssätze an sich*). An intuitive proposition contains at least one objective intuition, that is, a simple idea that represents exactly one object; a conceptual proposition contains no objective intuition. After Bolzano, philosophers dispensed with the distinction between conceptual and intuitive propositions. So why did Bolzano attach philosophical importance to it? I will argue that, ultimately, the value of the distinction lies in the fact that conceptual and intuitive truths have different objective grounds: if a conceptual truth is grounded at all, its ground is a conceptual truth. The difference in grounds between conceptual and intuitive truths motivates Bolzano's criticism of Kant's view that intuition plays the fundamental role in mathematics, a conceptual science by Bolzano's lights."
41. ———. 2022. "Grounding, Simplicity, and Repetition." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 301-318. New York: Oxford University Press
 "For Bolzano, grounding often goes along with a reduction of *propositional complexity*, where he takes the complexity of a proposition to depend not only on

- how many* ideas occur in it, but also by how *often* each of them occurs (so that the proposition *that Ann is wise* is less complex than the proposition *that Ann isn't unwise*). But this raises the Repetition Problem, which Mark Textor explores in his paper: how can a whole contain one and the same entity more than once?" (p. 37)
42. Thompson, Paul B. 1981. "Bolzano's Deducibility and Tarski's Logical Consequence." *History and Philosophy of Logic* no. 2:11-20
Abstract: "In this paper I argue that Bolzano's concept of deducibility and Tarski's concept of logical consequence differ with respect to their philosophical intent. I distinguish between epistemic and ontic approaches to logic, and argue that Bolzano's deducibility presupposes an epistemic approach, while Tarski's logical consequence presupposes an ontic approach."
43. Tolley, Clinton. 2012. "Bolzano and Kant on the Place of Subjectivity in a *Wissenschaftslehre*." *Grazer Philosophische Studien* no. 85:63-88
Summary: "Throughout his career, Bolzano presents his account of knowledge and science as an alternative to 'the Critical philosophy' of Kant and his followers. The aim of this essay is to evaluate the success of Bolzano's own account—and especially, its heavy emphasis on the objectivity of cognitive content—in enabling him to escape what he takes to be the chief shortcomings of the 'subjective idealist philosophy'. I argue that, because Bolzano's own position can be seen to be beset by problems that are both recognizably similar to, and possibly even worse than, those that he takes to afflict Kant's account of the elements of our knowledge, Bolzano's attempt to fully overcome the alleged vices of Kant's idealism by 'extruding' semantic content from the mind must be judged to be less than satisfactory."
44. ———. 2013. "Bolzano and Kant on the Nature of Logic." *History and Philosophy of Logic* no. 33:307-327
Abstract: "Here I revisit Bolzano's criticisms of Kant on the nature of logic. I argue that while Bolzano is correct in taking Kant to conceive of the traditional logic as a science of the activity of thinking rather than the content of thought, he is wrong to charge Kant with a failure to identify and examine this content itself within logic as such. This neglects Kant's own insistence that traditional logic does not exhaust logic as such, since it must be supplemented by a transcendental logic that will in fact study nothing other than thought's content. Once this feature of Kant's views is brought to light, a much deeper accord emerges between the two thinkers than has hitherto been appreciated, on both the nature of the content that is at issue in logic and the sense of logic's generality and formality."
45. ———. 2014. "Bolzano and Kant on Space and Outer Intuition." In *New Anti-Kant*, edited by Lapointe, Sandra and Tolley, Clinton, 157-191. London: Palgrave Macmillan
"Challenges to Kant's account of geometry appear already in some of Bolzano's earliest publications (cf. Bolzano 1810), and are developed more sustainedly in his later discussions of Kant in the 1837 *Wissenschaftslehre* ('WL') and those recorded by Přihonský in the 1850 *New Anti-Kant* ('NAK'). Bolzano argues, against Kant, that it is possible to define the representation of space through mere concepts alone, without this definition including any representations whatsoever drawn from intuition (cf. WL §79.6, I.366; §79 *Anm*, I. 369–370; NAK 74). In this respect, Bolzano thereby puts forward a form of geometrical 'logicism' *avant la lettre*.(4) In fact, Bolzano's criticisms go considerably further, insofar as he argues that the very idea of a pure intuition is essentially incoherent (as we will see below, cf. §§4–5). Yet while existing treatments of Bolzano's criticism of Kant on space have focused primarily on Bolzano's contrasting account of knowledge in geometry and mathematics more broadly, much less attention has been paid to the consequences that Bolzano's rejection of pure intuition has for Bolzano's own account of our intuitions of external objects – representations that Bolzano himself also calls 'outer intuitions'.(5) This will be my focus in what follows." (p. 158)
(4) Cf. Coffa 1991, 27f.; Sebestik 2003, 54f.; cf. Palagyi 1902, iii.
(5) An early start on this topic can be found in Palagyi 1902, chapter VI (esp.

§18). Some more recent helpful treatments of related topics can be found in George 2003 and Rosenkoetter 2012. For a discussion of Bolzano's rejection of Kant's doctrine of the pure intuition of time that is in key ways complementary to what follows, see George 1987.

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46. Trlifajová, Katerina. 2018. "Bolzano's Infinite Quantities." *Foundations of Science* no. 23:681-704

Abstract: "In his *Foundations of a General Theory of Manifolds*, Georg Cantor praised Bernard Bolzano as a clear defender of actual infinity who had the courage to work with infinite numbers. At the same time, he sharply criticized the way Bolzano dealt with them.

Cantor's concept was based on the existence of a one-to-one correspondence, while Bolzano insisted on Euclid's Axiom of the whole being greater than a part. Cantor's set theory has eventually prevailed, and became a formal basis of contemporary mathematics, while Bolzano's approach is generally considered a step in the wrong direction. In the present paper, we demonstrate that a fragment of Bolzano's theory of infinite quantities retaining the part-whole principle can be extended to a consistent mathematical structure.

It can be interpreted in several possible ways. We obtain either a linearly ordered ring of finite and infinitely great quantities, or a partially ordered ring containing infinitely small, finite and infinitely great quantities. These structures can be used as a basis of the infinitesimal calculus similarly as in non-standard analysis, whether in its full version employing ultrafilters due to Abraham Robinson, or in the recent "cheap version" avoiding ultrafilters due to Terence Tao."

47. van der Schaar, Maria. 2007. "Bolzano on Judgement and Error." In *The Logica Yearbook 2006*, edited by Tomala, O and Honzi, R., 211-221. Prague: Filosofia
- "Keeler (1934) ends his history of the problem of error with Kant, and Balduin Schwarz, in his article on 'Irrtum' in the *Historisches Wörterbuch der Philosophie*, only mentions 'the important analysis' of error given by Bolzano. In the less known third part of the *Wissenschaftslehre* (1837), the 'Erkenntnislehre', there are several chapters on judgement, knowledge and truth, with a special section on error. Besides the logical / conceptual question how error is possible, Bolzano also asks the epistemological / psychological question what the causes of error are, how error arises in us.

With respect to the concept of error, one has to distinguish between act and product. 'Error' and the German term 'Irrtum' stand for the product, resulting from an act of erring ('das Irren'). The distinction is a special case of the distinction between the act of judgement and the judgement product. Both act and product need to be distinguished from the proposition, which Bolzano also calls an error, if it is false but held true.

Because Bolzano explains error primarily as incorrect judgement (WL, I, § 36), the question what judgement is comes first (section 2). To understand the concept of error, one also needs to understand what knowledge is (section 3).

In my analysis of Bolzano's notions of judgement and knowledge I have profited from Mark Siebel's two recent articles on these topics (Siebel, 1999 and 2004). In

section 4 Bolzano's concept of error will be dealt with." (p. 212)

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48. Waldegg, Guillermina. 2001. "Ontological Convictions and Epistemological Obstacles in Bolzano's Elementary Geometry." *Science and Education* no. 10:409-418
 Abstract: "Bernard Bolzano (1781-1848) was a contemporary of the founders of non-Euclidean geometry and of the renovation of projective geometry. However, he did not participate in the movement transforming concepts and methods which crystallized in a new order of geometry at the beginning of the nineteenth century. On the contrary, throughout his life Bolzano tried to demonstrate Euclid's postulate of parallel lines.
 Two ontological convictions played the role of epistemological obstacle for Bolzano and prevented him even from imagining the possibility that non-Euclidean geometries might exist. In the first place, Bolzano thought that Euclidean geometry had an intrinsic structure and thus geometrical space must be intrinsically Euclidean. Secondly, the description of this structure contained the existence of an "objective" connection between geometrical truths; a basic truth was, by its nature, "simple and general".
 This article forms part of the body of work aimed at identifying obstacles in the history of mathematics in order to confront them with obstacles to learning and to establish their epistemological character."
49. ———. 2005. "Bolzano's Approach to the Paradoxes of Infinity: Implications for Teaching." *Science & Education* no. 14:559-577
 Abstract.: "In this paper we analyze excerpts of *Paradoxes of the Infinite*, the posthumous work of Bernard Bolzano (1781–1848), in order to show that Georg Cantor's (1845–1918) approach to the problem of defining actual mathematical infinity is not the most natural. In fact, Bolzano's approach to the paradoxes of infinity is more intuitive, while remaining internally coherent. Bolzano's approach, however, had limitations. We discuss implications for teaching, which include a better understanding of the responses of students to situations involving actual mathematical infinity, for it is possible to draw a kind of parallel between these responses and Bolzano's reasoning."
50. Wedberg, Anders. 1984. "Perfection and Innovation: Bernard Bolzano." In *A History of Philosophy. Vol. 3: From Bolzano to Wittgenstein*, 51-85. Oxford: Oxford University Press.
51. Winner, Thomas G. 1994. "Peirce and Bolzano." In *Living Doubt. Essays Concerning the Epistemology of Charles Sanders Peirce*, edited by Debrock, Guy and Hulswit, Menno, 157-169. Dordrecht: Reidel
 "Like Peirce, whom he preceded by roughly half a century, Bernard Bolzano (1781–1848), the brilliant mathematician, logician and semiotician who taught and wrote in Prague, was little recognized in his lifetime. Like Peirce, he endured persecution for his uncompromising attitudes, in his case both in science and political-religious life: also Bolzano's teaching career, like Peirce's, was cut short, in Bolzano's case because of official displeasure of the Vatican and the Vienna court over his resolute and unwavering liberalism in religious, social and political matters and towards the relation of Czechs and Germans in the Bohemian crownlands of the Austro-Hungarian monarchy. Bolzano's principal scientific contribution was, like Peirce's, in the area of mathematics and logic; and Bolzano's logic, like Peirce's, contained major contributions to semiotics, which Bolzano called the theory of signs (*Zeichenlehre*) and *Semiotik*, though Bolzano's *Zeichenlehre* was certainly not as

comprehensive and systematic as Peirce's semeiotic. Unlike Peirce, Bolzano is known primarily to logicians and to specialists in Catholic theology, while his semiotics has received relatively little attention." (p. 157)

52. Wrinch, Dorothy Maud. 1917. "Bernard Bolzano (1781-1848)." *The Monist. An International Quarterly Journal of General Philosophical Inquiry* no. 27:83-107 "In Bolzano we find the virtues of human sympathy and insight coupled with the austerer virtues of the metaphysician and logician. He was a man of action as well as a man of ideas. He was well known for his kindly disposition and his broadmindedness. He possessed not only the sympathy with the poor necessary for a social reformer, but the ability to develop his ideas of social reconstruction on practical lines. Not only did he elaborate a theory of an ideal state, but he also introduced numerous reforms in the actual state of which he was a member. He studied theology very earnestly as a young man and later wrote a great deal on the subject. Even though his liberal views brought him into collision with those on whom his livelihood depended, yet he courageously continued his teaching and writing, always making it his aim to seek for truth. He was a metaphysician of some importance and his treatises on metaphysics are valuable, not only for the original thought which they contain, but also for his important criticisms of Kant. In esthetics his work is by no means without interest, and to the psychology and ethics of his day he made very valuable contributions. But preeminently he was a mathematician and logician." (p. 83)