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Selected bibliography on Bernard Bolzano's Contributions to Logic and Ontology. Fourth Part: M - R

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Bernard Bolzano. Annotated Bibliography on His Practical Philosophy

Studies on Bolzano's Logic and Ontology

1. Malink, Marko. 2022. "Aristotle and Bolzano on Grounding." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 221-243. New York: Oxford University Press
"Marko Malink examines Aristotle's conception of scientific proofs as the historical roots of Bolzano's conception of grounding and compares the two philosophers' views on infinite grounding chains, on the role that generality plays in ground-revealing proofs, and on scientific knowledge." (p. 37)

2. Mancosu, Paolo. 1999. "Bolzano and Cournot on Mathematical Explanation." *Revue d'histoire des sciences* no. 52:429-456
Abstract: "Recent discussions on the topic of « mathematical explanation » have focused on the distinction between explanatory and non-explanatory proofs. The former proofs are supposed to differ from the latter in that they not only establish that a result is true but also show why it is true. This opposition is at the core of the philosophies of mathematics of Bolzano and Cournot. The paper analyzes Bolzano's theory of Grund and Folge, and Cournot' s opposition between the logical and the rational order, emphasizing their relevance to the issue of mathematical explanation. The final part of the paper investigates the shortcomings of Bolzano's and Coumot's theories as explications of mathematical explanation."

3. Mates, Benson. 1992. "Bolzano and Ancient Pyrrhonism." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 121-139. Firenze: Leo S. Olschki
"Bolzano's attempt to refute so-called "radical" or "complete" skepticism is carefully described in Professor Berg's introduction to his edition of the *Wissenschaftslehre* (WL). Two forms of such skepticism are there distinguished. The thesis of the *ontological* form is
(1) No propositions (*Sätz an sich*) is true and that of the *epistemological* form is
(2) No judgment (*Urteil*) is true.
Bolzano's principle arguments against these are roughly as follows. Against (1) he argues that, for any proposition S, either S is true or the proposition that S is false is true. Therefore, at least one proposition is true. The argument against (2) is less clear. Bolzano (WL 40) takes the problem to be that of convincing a radical skeptic that, after all, he must recognize the truth of at least one proposition. After considering various possibilities, he concludes that the skeptic will have to accept as true at least the proposition that he has ideas (*Vorstellungen*), for obviously he confirms this proposition the moment he doubts or denies it. The point, I suppose, is that, just as one cannot doubt that there are men on the moon if one has no idea of what it is to be a man or to be on the moon, so the skeptic, if he has no ideas, is in no position to doubt anything, not even that he has ideas. Bolzano thinks that while the skeptic might refuse publicly to admit the proposition in question, "nevertheless he will surely feel in his innards that it is true... and if he feels this, we have won". Whatever one may think of these arguments, in this paper I am not concerned to evaluate them but only to consider whether they refute Pyrrhonism, as Bolzano seems to suppose." (pp. 121-122)
(...)
The root of Bolzano's failure to appreciate the force of Pyrrhonism is, in my opinion, that he does not realize that its self-referential aspect is essential. This aspect is not something that Sextus is reluctant to admit but is rather a feature that he emphasizes over and over again and that he obviously regards as crucial to the

consistency of the skeptic's position. Bolzano's failure to understand this is especially evident at WL 40, where he quotes and discusses one of the many passages in which Sextus points out the self-reference of the skeptic's slogans (*phonai*), i.e., pronouncements like "contrary claims are equal", "no more this than that", "I decide nothing", etc. Bolzano says:
In setting forth the various formulae with which the skeptic is accustomed to express his state of doubt, Sextus Empiricus tries to employ maximal caution so as to protect it from the charge of self-contradiction, but nevertheless he finds himself compelled at the end to admit
"As concerns all the skeptic slogans the following must be understood in advance, namely that we do not maintain their truth in any absolute way, since we say that they themselves are included among the things to which they apply -- just as cathartic drugs do not merely eliminate humor from the body but also expel themselves along with the humors" (*Outlines of Pyrrhonism* I 206).
"This amounts to the reluctant admission", says Bolzano, that the skeptic ceases to be a skeptic as soon as he declares himself to be a skeptic. Only if he keeps silent and makes no judgment, not only in words but also internally, is he a complete doubter; and as long as this condition exists we *others* can say of him truly that he doesn't know a single truth. But as soon as he *himself* says it, the condition ceases and his judgment is therefore false.
But there is no "reluctant admission" here, and the Pyrrhonist doesn't have to be silent if he is to remain a Pyrrhonist. He will say "*It seems to me now that* contrary claims are equal" and "*It seems to me now that* there is no more reason for this than that", and so on. What he refrains from are flat out categorical statements, whether concerning his own skepticism or anything else.
It will be evident that this form of skepticism is not easily refuted. Since the Pyrrhonist agrees only to propositions expressing what *seems* to him at the moment to be the case, it is even unclear what a refutation would be like. But that is a topic for another day". (pp. 138-139).

4. Morscher, Edgar. 1986. "Was Existence Ever a Predicate?" *Grazer Philosophische Studien* no. 25/26:269-284
Abstract: "The question "Was 'existence' ever a predicate?" in a way already suggests its own answer, that this is really the wrong question to ask, because 'existence' has always been a predicate. Even those, such as Kant, who supposedly opposed this view, in fact held it. They merely denied that 'existence' is a "normal" first-order predicate. Not only Kant, but also Bolzano, Frege and Russell claimed that it is a second-order predicate. There is substantive disagreement between Kant and Bolzano on the one hand and Frege and Russell on the other over two issues: the former claim that this second-order predicate applies to no concept analytically and that it can be properly ascribed to a singular concept, whereas the latter deny both of these claims."

5. ———. 1986. "Propositions and States of Affairs in Austrian Philosophy before Wittgenstein." In *From Bolzano to Wittgenstein. The Tradition of Austrian Philosophy*, edited by Nyiri, Janoc Cristof, 75-85. Vienna: Hölder-Pichler-Tempsky.

6. ———. 1987. "Propositions and All That: Ontological and Epistemological Reflections." In *Logos and Pragma. Essays on the Philosophy of Language in Honour of Professor Gabriël Nuchelmans*, edited by Rijk, Lambertus Marie de and Braakhuis, Henk A.G., 241-257. Nijmegen: Ingenium Publishers
"Bernard Bolzano was one of the first philosophers in modern times to develop explicitly a complete theory for entities like propositions, statements and states of affairs. I will first describe and clarify the main features of his theory, and then sketch the subsequent development to our day." (p. 243)
(...)
"Let me now complete my historical sketch. Up to now I have only discussed Bolzano's doctrine of propositions. I concentrated on Bolzano's doctrine because I think that he gave the clearest account, the clearest description of propositions

available in his time, and that none of the philosophers who followed, including Frege, has made an essential improvement in this respect. Although Bolzano's doctrine, his description of the propositions and the ontological status he ascribes to them, is far from being satisfactory, because it is insufficiently clear, no other philosopher up to our time has done any better. I have therefore explained Bolzano's doctrine in more detail in order to have one representative traditional doctrine to which I can refer in what follows.
What seems very interesting to me and what I have always been very impressed by is the fact that philosophers with completely different backgrounds and from different schools developed, at the same time as Bolzano and afterwards, quite similar views, sometimes using almost the same words as Bolzano, without being familiar with his work. Although this is far from being a proof for the truth of his doctrine, it is nevertheless a fact a philosopher cannot pass by because it indicates that this is not the doctrine of an eccentric outsider. On the contrary, it has attracted many philosophers, including such prominent ones as Frege, Wittgenstein and Russell, Husser! and Meinong, Windelband and Rickert. (I have described the views of some of these philosophers and compared them in another paper: Morscher (1972 [*Von Balzano zu Meinong: zur Geschichte des logischen realismus*])." (p. 248)

7. ———. 1997. "Bolzano's Method of Variation: Three Puzzles." *Grazer Philosophische Studien* no. 53:139-165
Abstract: "Bernard Bolzano's most fruitful invention was his method of variation. He used it in defining such fundamental logical concepts as logical consequence, analyticity and probability. The following three puzzles concerning this method of variation seem particularly worth considering. (i) How can we define the range of variation of an idea or the categorial conformity of two ideas without already using the concept of variation? This question was raised by Mark Siebel in his M.A. thesis. (ii) Why must we define analyticity by means of (simultaneous or successive) variation of several ideas rather than by means of replacing a single idea? This problem is suggested by an example due to W.V.O. Quine, John R. MyhiII and Benson Mates. (iii) Must every 'there is ...' sentence be synthetic for Bolzano, as his pupil Franz Prihonsky claims in his booklet *Neuer Anti-Kant,* or can a `there is...' sentence be logically analytic?"

8. ———. 2006. "The Great Divide within Austrian Philosophy. The Synthetic a Priori." In *The Austrian Contribution to Analytic Philosophy*, edited by Textor, Mark, 250-263. New York: Routledge
"In this chapter I will try to show that the divergent Austrian ways of being anti-Kantian do not vanish even when we focus on this single topic. To illustrate this view, I will take as my examples Bernard Bolzano and Rudolf Carnap, who both belong – for different reasons – to the so-called Austrian tradition in philosophy. Both are fully conversant with Kant's work, and both have a critical attitude toward it and are in this sense anti-Kantian. This is also true when it comes to the question of the synthetic a priori: both refute strongly Kant's treatment of the synthetic a priori. However, whereas Carnap denies synthetic sentences a priori altogether, Bolzano does not deny their existence but only the way in which Kant justifies their truth.
What is even more important is that Bolzano not only – contrary to Carnap – accepts Kant's synthetic a priori, but even extends it to the realm of logic. In clear opposition to Kant and Carnap, who take all logical truths to be analytic, there are synthetic truths for Bolzano even in the area of logic. I will try to argue for this claim in the following sections." (p. 250)

9. ———. 2008. *Bernard Bolzano's Life and Work*. Sank Augustin: Academia Verlag
Table of Contents: Preface 9; Introduction 13; 1. Bolzano's Life and Scientific Career 17; 2. Bolzano's Removal from Office and the "Bolzano Trial" 23; 3. A Short Survey of Bolzano's Work 29; 4. Logic 33; 5. Epistemology and Philosophy of Science 75; 6. Ethics 89; 7. Aesthetics 107; 8. Political and Social Philosophy

113; 9. Philosophy of Religion and Theology 125; 10. Metaphysics 135; 11. Philosophy of Nature and of Physics 139; 12. Philosophy of Mathematics 141; 13. Metaphilosophy and History of Philosophy 149; 14. The So-called Bolzano Circle and Bolzano's Influence on the Development of the Sciences and on Intellectual History 151; Appendix: A Formal Reconstruction of Bolzano's Method of Idea-Variation and of his Definitions of Logical Truth and of Logical Consequence 159; Bibliography 169; Index of Names 207-211.
"Despite the enormous increase of interest in Bolzano's philosophy during the last decades, an up-to-date monograph on Bolzano's philosophy is still a desideratum. The last book that might be called a monograph on Bolzano's philosophy dates from almost 100 years ago; it is Shmuel Hugo Bergmann's *Das philosophische Werk Bernard Bolzanos* (Halle/S. 1909), written in the spirit of the Brentano school, in particular of Bergmann's teacher Anton Marty.
When I was invited by the Editors of the *Stanford Encyclopedia of Philosophy* to contribute the entry on Bernard Bolzano, I took it as a challenge for starting my long-standing plan to write a monograph on Bolzano's philosophy. The present book is, to be clear, merely the first step toward this end." (from the *Preface*)
(...)
"Bolzano's uncommonly versatile work culminated in three extensive main writings in three different areas of knowledge: 1) in theology his four volume *Textbook of the Science of Religion* (Bolzano 1834b), 2) in philosophy the four volume *Theory of Science* (Bolzano 1837a), which provides a new foundation for logic and is at the same time an extensive manual of logic, and 3) in mathematics the *Theory of Quantities*, conceived of as a monumental work, but not completed.
Bolzano's teaching was concerned exclusively with fundamental topics of theology, in addition he worked mainly in logic. Nevertheless, his scientific development began in mathematics. It was mathematics that was the starting point for his scientific work and to which he ultimately returned in order to create a new foundation on which mathematics as a whole could be built; he succeeded in doing this, however, only in bits and pieces." (p. 29)

10. ———. 2022. "The Grounds of Moral 'Truths'." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 343-363. New York: Oxford University Press
"Central to Bolzano's ethics is his Supreme Moral Law, i.e. an ethical truth that grounds all other ethical truths. While Bolzano considers this law to be fundamental in the realm of ethics, he also claims that it is not
an ungrounded, basic truth. Edgaqr Morscher discusses this view in the context of a succinct reconstruction of Bolzano's views on grounding, his ethics, and his deontic logic." (p. 38)

11. Morscher, Edgar, and Simons, Peter. 2014. "From Bolzano via Quine to Fine." In *Joint Ventures in Philosophy*, edited by Morscher, Edgar and Simons, Peter, 137-156. Sankt Augustin: Academia Verlag.

12. Mugnai, Massimo. 1992. "Leibniz and Bolzano on the "Realm of Truths"." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 207-220. Firenze: Leo S. Olschki
"In his article *Propositions and Sentences* (1956) Alonzo Church pointed out -- on the basis of a suggestion made by Joseph Maria Bochenski -- that strong analogies exist between Bolzano's theory of *Satz an sich* and Gregory of Rimini's doctrine of complexe significabile.' In the same essay, Church also pointed out that Bolzano appealed to Leibniz as to a logician who plainly recognized propositions in the abstract sense. After Church's essay, it became very usual to mention Gregory of Rimini in reference to Bolzano's ontological conceptions. Nevertheless, we do not have any evidence of a direct influence of Gregory of Rimini's ideas on Bolzano's philosophy of logic. Bolzano seems to have only a limited acquaintance with the logic of the late medieval period: the credit accorded to Savonarola's Compendium logicae - a standard work which is absolutely lacking in originality - corroborates, I

think, this view.2 Yet Bolzano may have benefited by late scholastic inheritance through the intermediation of later works, like those of Campanella, Clauberg, Fonseca, Keckermann, Leibniz and Wolf.' In fact, as already mentioned, Leibniz is the first author whom Bolzano explicitly refers to, in paragraph 21 of the Wissenschaftslehre, as a forerunner of the Satz an sich theory:
"Thus Leibniz uses as equivalent the expressions *proposition* and *cogitatio possibilis (Dial. de Connexion inter Verba a Res* [C. I. Gerhardt, ed. *Philos. Schriften, vol.* VII, p. 190]. This obviously presupposes that by propositions he meant propositions in themselves." (*)
The Leibniz's work on which Bolzano explicitly bases this conviction is the *Dialogus de connexione inter res et verba*, first published by Raspe in 1765 -- a work whose content paradoxically seems to partly disprove Bolzano's interpretation.' Thus Church considers it «
"an exaggeration or a misunderstanding" on Bolzano's part to have attributed to Leibniz's D*ialogus* "the use of the word propositio for proposition in the abstract sense" or *Satz an sich*.(6) The same remarks are repeated by Prof. Berg in his monograph on Bolzano's logic: after having identified Bolzano's *Satz an sich* with Frege's *Gedanke*, Prof. Berg writes:
According to Leibniz a proposition (*propositio*) is a possible thought (*cogitatio possibilis*), which is capable of being true or false... But no thought or reasoning is possible without words or some other kind of signs. And under transformation of a proposition into a different language a certain relationship (*proportio*) among the signs and between the signs and the objective reality is transformed into a similar relationship. The last two conditions fit Aristotle's and Peter of Spain's but not Frege's notion of proposition. Therefore... it must have been a misunderstanding on Bolzano's part to have attributed to Leibniz the use of the world "propositio" for *Satz an sich.*(7)
In what follows, I intend to take up the problem of the correcteness of the interpretation given by Bolzano and then to develop a comparison between the positions of Leibniz and those of Bolzano relative to the notions of idea, proposition and truth.
In the notes I have employed the following abbreviations: WL = B. Bolzano, *Wissenschaftslehre*, in B. Bolzano, Gesamtausgabe, Reihe I, Schriften, Stuttgart-Bad Cannstatt, Friedrich Frommann Verlag 1985 ff; GP = G. W Leibniz, *Die philosophische Schriften*, Hrsg. von C. I. Gerhardt, Berlin, Akademie, 1857-90, vol. I-VII; VE = G. W. Leibniz, *Vorausedition zur Reihe VI - Philosophische Schriften* - Munster, Akademie, 1982 ff.
(1) A. Church, *Propositions and Sentences*, in I. M. Bochenski, A. Church, N. Goodman, *The Problem of the Universals*, Notre Dame, Notre Dame Press, 1956, p. 3.
(2) WL 1, 11/1, pp. 105 ff.
(3) WL 1, 11/1, pp. 234 ff.
(4) WL 1, 11/1, p. 111.
(5) Cfr. *Oeuvres philosophiques latines et francoises de feu Mr. de Leibnitz ...* publiees par Mr. Rud. Eric Raspe, Leipzig, 1765, pp. 505-512.
(6) A. Church, *op. cit*., p. 10.
(7) J. Berg, *Bolzano's Logic*, Stockholm, Almquist and Wiksell, 1962, pp. 51-52.
(*) [cited in German in the original; I cite from the translation of *Wissenschaftslehre* by Rolf George, p. 24]

13. Mulligan, Kevin. 2022. "Logic, Logical Norms, and (Normative) Grounding." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 244-275. New York: Oxford University Press
"While Bolzano's writings were largely ignored by the philosophical community of his time, they later aroused the attention of Franz Brentano and his students, in particular that of Edmund Husserl. Kevin Mulligan examines a range of Husserl's views on grounding and their relation to Bolzano's views. A particular emphasis is

laid on Husserl's conception of logic: is logic a normative or a theoretical discipline? Relatedly:
what is the connection between logical norms and logical truths? Husserl argues that logic is a theoretical discipline and that logical truths ground logical norms. In order to understand and evaluate this view, it is compared with Bolzano's account of the grounds of moral truths." (p. 37)

14. Neeman, Ursula. 1970. "Analytic and Synthetic Propositions in Kant and Bolzano." *Ratio* no. 12:1-25
"Whereas Kant regards the structure of being and knowing as identical, Bolzano interprets the Kantian true synthetic propositions as true propositions, in which the predicate is a characteristic of the subject and not a component of the notion of the subject (characteristic =df. a property of the object, which falls under the concept; component =df. ingredient of the concept). These propositions are analytic in a wider sense, because they render possible an analysis of an object, whereas the logico-analytic propositions render possible only an analysis of their concept. Therefore Bolzano also distinguishes between deductibility (ordo cognoscendi) and ground-consequence relation (ordo essendi) and grounds the latter on the principle of simplicity. A discovery of an objective connection in mathematics is only possible by a strict determination of the basic concepts and by axiomatization, because in opposition to Kant, Bolzano thinks mathematical laws to be discoveries and not creations of the human mind."

15. Otte, Michael. 2008. "Proof and Explanation from a Semiotical Point of View." *Relime*:23-43
Abstract: "A distinction between proofs that prove and proofs that explain has over and again played an important role within recent discussions in epistemology and mathematics education.
The distinction goes back to scholars who, like Bolzano or Dedekind, have tried to reestablish pure mathematics as a purely conceptual and analytical science. These endeavors did in particular argue in favor of a complete elimination of intuitive or perceptual aspects from mathematical activity, arguing that one has to rigorously distinguish between a concept and its representations. Using a semiotical approach which negates such a separation between idea and symbol, we shall argue that mathematics has no explanations in a foundational sense. To explain amounts to exhibiting the meaning of something.
Mathematics has, however, as we shall try to show, no definite meanings, neither in the structural intra-theoretical sense nor with respect to intuitive objectivity. Signs and meanings are processes, as we shall argue along with Peirce."
"Before we can address the issue of proof and explanation we have to get rid of traditional *Bewusstseinsphilosophie* (philosophy of consciousness), that is, popularly speaking, the belief that “meanings are in the head” and knowledge is some sort of mental experience. After Kant epistemology began to ramify and various new philosophies of mathematicsarose in which meaning, rather than mind played the central role. But the view that there exists an epistemologically autarkic or self-sufficient epistemic subject, which serves itself from external sensations and internal experiences or representations (Vorstellungen) to thereby constitute true knowledge, is a myth and should also be abandoned.
In Part I of this paper we try to provide some pertinent arguments to this end, based on Peirce’s semiotics.
“Consciousness is used to denote the I think, the unity of thought; but the unity of thought is nothing but the unity of symbolization” (Peirce CP 7.585). Part II treats the questions of proof and explanation with respect to the ideas of Bolzano on the one hand and Peirce on the other. Part III presents some examples and tries to make a connection with current debates about the issue in mathematical education and cognitive psychology." (p. 25)

16. ———. 2009. "The Analytic/Synthetic Distinction in Kant and Bolzano." In *Relatively and Philosophically E[a]rnest. Festschrift in Honor of Paul Ernest's 65*

Birthday, edited by Sriraman, Bharath and Goodchild, Simon, 39-56. Missoula: Information Age Publishing.

17. Parsons, Charles. 2012. "Two Studies in the Reception of Kant's Philosophy of Arithmetic." In *From Kant to Husserl: Selected Essays*, 80-99. Harvard: Harvard University Press
"The present essay takes its point of departure from a thought I have had at various times in thinking about interpretations of Kant's philosophy of mathematics in the literature, in particular that offered by Jaakko Hintikka. That was that if the interpretation is correct, shouldn't one expect that to show in the way that Kant's views were understood by others in the early period after the publication of the first *Critique*? That reflection suggests a research program that might be of some interest, to investigate how Kant's philosophy of mathematics was read in, say, the first generation from 1781. I have not undertaken such a project. However, I will make some comments about two examples of this kind. In doing so I haven't always kept my eye 0n Kant, because the figures involved are of interest in their own right. The first is Johann Schultz (1739-1805), the disciple of Kant who was professor of mathematics in Köinigsberg.Thc second is Bernard Bolzano (1781-1848), who in an early essay of 1810 (*)
offered a highly critical discussion of Kant's theory of construction of concepts in intuition. In one way, I think the result of this little experiment is negative, in that it does little toward settling disputed questions about the interpretation of Kant. On the other hand, I think it brings out some problems of Kant's views that could be seen either at the time he wrote or not long after." (p. 80)
(*) [*Beyträge zu einer begründeteren Darstellung der Mathematik = Contributions to a Better Grounded Presentation of Mathematics*]

18. Poggiolesi, Francesca. 2022. "Bolzano, (the Appropriate) Relevant Logic, and Grounding Rules for Implication." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 319-342. New York: Oxford University Press
"While Bolzano sharply distinguishes grounding from logical deducibility {in modern terminology: entailment), he also regards the two notions as importantly connected. He sees a particularly close connection
between grounding and exact deducibility (a special case of deducibility). Francesca Poggiolesi examines this latter notion and compares it to notions of relevant entailment. She argues that Neil Tennant's system CR is the best model for Bolzano's ideas, and can in turn also serve as a framework for developing grounding rules for conditionals." (p. 37)

19. Proust, Joëlle. 1981. "Bolzano's Analytic Revisited." *The Monist. An International Quarterly Journal of General Philosophical Inquiry*:214-230
"What I propose is to reconsider the interpretation of Bolzano's concept of analytic propositions which was offered thirty years ago by Bar-Hillel.(1) The claim of Bar-Hillel was that, in a late addition to his book, *The Theory of Science*,(2) Bolzano actually had been radically improving his concept of analyticity, thus creating some inconsistencies with the previous, uncorrected version. This allows us to equate the new Bolzanian definition of analytic with what was to be defined, a century later, as logical truth by W. V. Quine. Bar Hillel's interpretation has been uncritically accepted by commentators, although the historical issue has been rightly challenged by J. Berg. What I want to show is that, in spite of a surface analogy between Bolzano's phrasing of the definition of 'logical analytic' and Quine's definition, certain considerations should lead us to call that parallel into question. Attractive as it may be for a Quinian, such a view of Bolzano's analytic can be shown as incompatible with the leading ideas of his philosophy of logic. Furthermore, there is enough evidence in other sections of the *Theory of Science* to show that Bolzano's criterion of analyticity is grounded on purely semantical properties and is part of a general account of logical properties in terms of the mapping of propositions to corresponding models." (p. 214)

(1) "Bolzano's Definition of Analytic Propositions," *Theoria* 16 (1950): 91-117 and *Methodos* 2, (1950): 32-55; reprinted in *Aspects of Language* Magnes Press 1970, pp. 3-32.
(2) Bernard Bolzano, *Wissenschaftslehre* (Sulzbach 1837, Leipzig 1914); partly translated by Rolf George: *Theory of Science*, (Oxford: Basil Blackwell, 1972). We shall quote this translation whenever available.

20. ———. 1989. *Questions of Form: Logic and the Analytic Proposition from Kant to Carnap*. Minneapolis: University of Minnesota Press
Translated by Anastasios Albert Brenner from the original French: *Questions de forme. Logique et proposition analytique de Kant à Carnap* - Paris, Fayard, 1986.
Section Two: *Bolzano's Renovation of Analiticity,* pp. 49-108.
"The specifically Bolzanian concept of analyticity is brought in at an advanced life, as the maturely formulated answer to a problem that never ceased to appear under different aspects. Only in the *Wissenschaftslehre* of 1837 does what we might call a "revolution" in analyticity occur. Earlier texts strive to adapt the Kantian definition so that it satisfies the new requirements of the anticritical mathematicians. But this definition, often revised, gives rise to growing difficulties. There were so many reasons for abandoning it, but also so many constraints working to shape the new definition. "Revolution," we said; but until the *Wissenschaftslehre*, analyticity was a marginal theme. Its main function was, as in Kant, to reveal the problematic existence of the synthetic a priori. From a theme of preliminary exposition, analyticity becomes in the work of 1837 an "integrated" concept: henceforth it is part of a philosophy and becomes inseparable from a method of identifying logical objects, variation. But this was a "Ptolemaic," not a "Copernican," revolution: instead of statically emphasizing the synthetic a priori, it becomes a notable property of certain propositions whose definition now requires a preliminary examination of other properties such as truth and validity. This definition, however, does not have a purely descriptive interest; the theses of Volume 3 must be taken seriously in order to portray with perfect clarity the deep interest that Bolzano had in his new definition of analyticity." (p. 49)

21. ———. 1999. "Bolzano's Theory of Representation." *Revue d'Histoire des Sciences* no. 52:363-383
Abstract: "Bolzano's theory of representation is one of the most radically intensionalist approaches to representation. It is based on the following three claims: A). A representation is essentially independent of thought and of linguistic expression; B). A representation is structured; C). Such a structure is independent of the objects represented. These claims are both tools and constraints relative to Bolzano's substantive goals. Bolzano ultimately aimed to carry out a deep transformation of mathematical and scientific practice, thanks to a more accurate conception of logic and of the role of logic in scientific exposition. I examine some of the consequences of Bolzano's claims in regard to his conception of mathematical treatises."

22. Roberts, Mark. 1994. "The Bearer of Truth and Falsity." *Southwest Philosophy Review* no. 10:59-67
Abstract: "Until Bolzano nearly all philosophers believed that truth and falsity are predicated of judgments of beliefs. Bolzano and other philosophers after him argue that propositions are the bearers of truth and falsity and that propositions have a timeless ideal existence: a position which seems to discredit completely their view that propositions are the bearers of truth and falsity. Yet, several arguments can be offered which show that propositions are the bearers of truth and falsity without introducing as a premise the timeless existence of propositions."

23. Rohloff, Waldemar. 2012. "From Ordinary Language to Definition in Kant and Bolzano." *Grazer Philosophische Studien* no. 85:131-149
Abstract: "In this paper I discuss Kant's and Bolzano's differing perspectives on ordinary natural language. I argue that Kant does not see ordinary language as providing semantically organized content and that, as a result, Kant does not believe

that ordinary language is sufficiently well-developed to support philosophical analysis and definition. By contrast, for Bolzano, the content given in ordinary language are richly structured entities he calls 'propositions in themselves'. This contrast in views is used to explain Bolzano's criticism of Kant's belief that definition is impossible for philosophical concepts. It is also used to explain Bolzano's criticism of Kant's methods of exposition of philosophical concepts."

24. Rojszczak, Artur. 2005. *From the Act of Judging to the Sentence. The Problem of Truth Bearers from Bolzano to Tarski*. Dordrecht: Springer
Edited by Jan Wolenski.
Chapter 7.1: *Bernard Bolzano (I): Sentences in Themselves*, pp. 111-115.
"I shall not go into the details of the multiplicity of Bolzano's ideas and their particular influence on the history of semantics. I shall, as I have tried to do with respect to every issue in this study, concentrate on his ideas within the theory of science as it is related to the problem of the truth bearer. In the context of the theory of truth, it is worth noting that Bolzano's position during his times, i.e. in the first half of the nineteenth century, was quite unusual. Bolzano's influence on this century was provided by his notion of the objectivity of truth in a way that also remained standard for the next century. Furthermore, the theory which should guarantee the objectivity of truth was, for Bolzano, his theory of sentences in themselves. Only the semantics of the twentieth century sees Bolzano's theory of sentences in themselves as an anticipation of the contemporary notion of proposition. I shall, however, refer to his *Fundamentallehre* [Theory of Fundamentals], i.e. to the first sections of his *Theory of Science*, which deals with the existence of objective truth and with the possibility of its cognition. I shall omit some elements of this theory that are irrelevant to my purposes; for example, Bolzano's proof of the existence of truth, his proof of the existence of infinitely many truths or the argument for the cognition of truths. In this part of Bolzano's argumentation, he focuses on the problem of skepticism, making an attempt to prove the fundamentalist position in epistemology.(1) I shall take the liberty of presenting Bolzano's ideas as far as truth bearers are concerned as contrasted with the views of Brentano and Twardowski on the objectivity of truth which I shall present in the next sections." (p. 111)
(1) Bolzano 1837, par. 40–43.

25. Rojszczak, Artur, and Smith, Barry. 2003. "Truthmakers, Truthbearers and the Objectivity of Truth." In *Philosophy and Logic in Search of the Polish Tradition: Essays in Honour of Jan Wolenski on the Occasion of His 60th Birthday*, edited by Kijania-Placek, Katarzyna, 229-268. Dordrecht: Kluwer
"The aim of this paper is to show that the account of objective truth taken for granted by logicians at least since the publication in 1933 of Tarski 's 'The Concept of Truth in Formalized Languages' arose out of a tradition of philosophical thinking initiated by Bolzano and Brentano. The paper shows more specifically that certain investigations of states of affairs and other objectual correlates of judging acts, investigations carried out by Austrian and Polish philosophers around the turn of the century, formed part of the background of views that led to standard current accounts of the objectivity of truth. It thus lends support to speculations on the role of Brentano and his heirs in contemporary logical pbilosophy advanced by Jan Woleński in his masterpiece on the *Logic and Philosophy in the Lvov-Warsaw School* of 1989." (p. 229)

26. Rollinger, Robin D. 2004. "Austrian Theories of Judgment: Bolzano, Brentano, Meinong, and Husserl." In *Phenomenology & Analysis. Essays on Central European Philosophy*, edited by Chrudzimski, Arkadiusz and Huemer, Wolfgang, 257-284. Frankfurt: Ontos Verlag
Reprinted in: R. D. Rollinger, *Austrian Phenomenology. Brentano, Husserl, Meinong, and Others on Mind and Object*, Frankfurt: Ontos Verlag 2009, pp. 233-262.
"Introduction

In nineteenth century German philosophy it was among the prevailing views that mental phenomena were to be divided into three classes: thinking, feeling, and willing. In Austria, however, two of the towering philosophers, Bernard Bolzano and Franz Brentano, held that presentations (*Vorstellungen*) and judgments (Urteile) make up two distinct classes of mental phenomena. Moreover, both of these philosophers saw it as an important task to work out a theory of judgment in particular. It is accordingly no surprise that Brentano's two most outstanding pupils, Alexius Meinong and Edmund Husserl, developed theories of judgment, though their results were markedly different from those of their predecessors and from each other's. In the following the line of Austrian philosophy from Bolzano to Husserl will be traced by presenting an overview of the four theories indicated in the title. The topic under consideration in these theories, though apparently little more than a chapter in descriptive psychology, is of great significance because it gives us an intersection for issues in epistemology, ontology, and philosophy of logic." (p. 257)

27. Rootselaar, Bob van. 1970. "Bernard Bolzano." In *Dictionary of Scientific Biography, Vol. 2*, edited by Gillispie, Charles Coulston, 273-279. New York: Charles Scribner's Sons

"Bolzano planned to elaborate the methodology begun in his *Beyträge* and to develop it into a complete lheory of science, of which a treatise on logic was to form the cornerstone. From 1820 on, he worked steadily on it, and his four-volume treatise *Wissenschaftslehre* appeared in 1837. The plan of the W*issenschaftslehre* appears clearly from the following subdivision (see Kambartel, *Bernard Bolzano's Grundlegung der Logik*, pp. 14-17):

(I) Fundamental theory: proof of the existence of abstract truths and of the human ability to judge.

(2) Elementary theory: theory of abstract ideas, propositions, true propositions, and deductions.

(3) Theory of knowledge: condition of the human faculty of judgment.

(4) Heuristics: rules to be observed in human thought in the search for truths,

(5) Proper theory of science: rules to be observed in the division of the set of truths into separate sciences and in their exposition in truly scientific treatises.

The work did not induce a complete revision of science, as Bolzano hoped, but, on the contrary, remained unnoticed and did not exercise perceptible influence on the development of logic. Some of the

innovations in logic contained in the first two volumes did attract attention, as well as excessive praisenotably from Edmund Husserl and Heinrich Scholz (see Berg. op. cit.; Kambanel, op. cit.; and the literature

cited in them).

The rise of logical semantics, initiated by Alfred Tarski in the l 930's, has led to a revival of the study of Bolzano's logic in the light or modem logic (see Berg, op. cit.) and of his theory of an ideal language.

The heart of Bolzano's logic is formed by his concepts of (abstract) proposition (*Satz an sich*), abstract idea (*Vorstellung an sich*), truth, and the notions of derivability (*Ab!eirbarkei*t) and entailment (*Abfolge*)." (pp. 277-278)

28. ———. 1992. "Axiomatics in Bolzano's Logico-Mathematical Research." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 221-230. Florence: Leo S. Olschki

"A discussion of Bolzano's axiomatical considerations requires some care, because his idea of axiomatization differs considerably from axiomatics as it is currently understood.

His *Wissenschaftslehre* is testimony of his concern for the foundation of science in general and in particular of the theoretical sciences. Among the theoretical sciences mathematics is of special interest.

According to Bolzano, the mathematics of his time was based on shaky foundations, and one of his activities was directed toward correction of this situation.

On the other hand he certainly had the intention to recapture essentially the entire body of existing mathematics and present it in full accordance with his newly laid foundations. This is the reason why on several occasions he revised existing proofs of known mathematical theorems." (p. 221)

29. Rosenkoetter, Timothy. 2012. "Kant and Bolzano on the Singularity of Intuitions." *Grazer Philosophische Studien* no. 85:89-129
Abstract: "Kant and Bolzano agree that intuitions are non-accidentally singular, but each offers more than one explanation of why this is the case. One model, exemplified by Bolzano's explication of intuitions as “this”-representations, posits a type of representation which is such that it can only have one object. A very different explanation, prominent in Kant's Transcendental Aesthetic, has recourse to the fact that certain classes of objects (spaces and times) can have only one instance, and argues on this basis that some representations with those contents are singular. This paper surveys various versions of these two explanations and uses each philosopher's answers to shed light on the other's."

30. Roski, Stefan. 2013. "A priori Knowledge in Bolzano: Conceptual Truths and Judgements." In *Judgement and the Epistemic Foundation of Logic*, edited by Schaar, Maria van der, 101-132. Dordrecht: Springer
"According to Kant, a true judgement can be called a priori in case it can take place absolutely (*schlechterdings*) independent of experience. Propositions that are knowable in this way are called a priori propositions by him (Kant, [*Critique of Pure Reason*], 1787 B, 3–4)." (p. 101)
(...)
"[Bolzano] tried to *give* a satisfactory theoretical account of the notion of synthetic *a priori* proposition. Roughly speaking, he located Kant’s mistake in the attempt to introduce a distinction among propositions by means of a distinction among judgements. Bolzano reversed this order and aimed instead to explicate the valid core of what Kant tried to capture in epistemic terms entirely in objective, logical ones." (p. 101)
(...)
"Bolzano’s explication has two aspects, a logical and an epistemological one.
The logical aspect consists in drawing a precise and workable distinction in terms of non-epistemic notions. The epistemological aspect concerns the way in which Bolzano’s suggestion might work: What is his account of how one can come to know synthetic truths a priori?
While there have been investigations of Bolzano’s objective explication of the notion of a priori proposition (see Textor 1996, chapter 4), the epistemological details have never been examined in great detail.(2) The task of this chapter is thus to tell the epistemological story behind Bolzano’s objective explication.
I should note right from the beginning that the aim of the chapter is descriptive and historical. Primarily, I want to make sense of what Bolzano plausibly had in mind, rather than assess its intrinsic plausibility." (p. 102)
(2) An exception is Lapointe (2010).
References
Lapointe, S. 2010. Bolzano, a priori knowledge and the classical model of science. *Synthese* 174:263–281.
Textor, M. 1996. *Bolzanos propositionalismus*. Berlin/New York: Walter De Gruyter.

31. ———. 2017. *Bolzano's Conception of Grounding*. Frankfurt: Vittorio Klostermann
Contents: Preface IX; I. Introduction 1; 2. Objective truth, variation & truth-preservation 19; 3. Explanatory priority: Bolzano’s pure logic of grounding 55; 4. Simplicity and economy: Bolzano’s impure logic of grounding 109; 5. Bolzano’s logic of grounding and the logic of metaphysical grounding 215; 6. Conclusion 233; l ist of abbreviations 251; List of symbols, definitions, and principles 253; Bibliography 257; Index 267-269.
"Overview of the book

As each of the following chapters will be accompanied by a detailed overview of its content and line of argumentation, I will confine myself here to a brief overview of the main line of argumentation of the book.
At the core of Bolzano's theory of grounding lies a set of general principles that express properties the relation exhibits according to him. An analysis of these principles, their interrelation, and their role in Bolzano's methodology will form the main bulk of the book. It is heuristically useful to divide these principles into two classes. The first class contains principles that hold for every case of grounding, irrespective of any specific properties of the relata. These principles capture, as it were, minimal conditions an explanatory relation has to satisfy according to Bolzano. The second class consists of more specific principles that mostly apply only to truths from deductive or a priori sciences. Adapting a distinction by Kit Fine, I will call the former Bolzano's *pure logic of grounding* and the latter his *impure logic of groundin*g.(52) Before we can dive into the details of Bolzano's theory, we will have to gain some familiarity with the nuts and bolts of his logical framework. This will be done in Chapter Two. Chapter Three then discusses Bolzano's pure logic of grounding, while Chapter Four is concerned with the impure logic of grounding. Chapter Five wraps up and draws some connections to the recent debate on grounding. In what follows I will sketch the content of each of these chapters in a little more detail." (p. 16)
(52) Cf. (Fine [The Pure Logic of Ground. *Review of Symbolic Logic* 5(1) 1-25] 2012b). The justification for employing this distinction will be given further below.

32. ———. 2019. "Bolzano and Kim on Grounding and Unification." *Synthese* no. 196:2971-2999
Abstract: "It is sometimes mentioned that Bernard Bolzano's work on grounding anticipates many insights of the current debate on metaphysical grounding. The present paper discusses a certain part of Bolzano's theory of grounding that has thus far not been discussed in the literature. This part does not so much anticipate what are nowadays common assumptions about grounding, but rather goes beyond them. Central to the discussion will be a thesis of Bolzano's by which he tries to establish a connectionbetween grounding and (deductive) unification. The paper spells out this thesis in detail and discusses the assumptions on which it rests. Next to this mainly historical aim, the paper also presents reasons why philosophers who are not interested in the historical Bolzano should find the thesis interesting by relating it to a certain view on unification and explanation that has been put forward by Kim. A final part of the paper provides a critical evaluation of the thesis against the background of current accounts of grounding."
Reference
Kim Jaegwon (1994) Explanatory Knowledge and Metaphysical Dependence. *Philosophical Issues* 5:51–69-

33. ———. 2020. "Bolzano." In *The Routledge Handbook of Metaphysical Grounding*, edited by Raven, Michael J., 76-89. New York: Routledge
"This chapter provides an overview of Bernard Bolzano's views about grounding. On Bolzano's account, grounding is an objective priority relation among true propositions that has certain explanatory features.The chapter briefly highlights historical influences on Bolzano's account of grounding and subsequently provides an overview of the most important aspects of it. As we shall see, Bolzano's account resembles current accounts of metaphysical grounding in many respects and can thus easily be related to many positions in the current debate.This is going to be a main focus of this chapter. Apart from that, we shall investigate some Bolzanian ideas about grounding that differ from the current orthodoxy but may constitute interesting additions, challenges or inspirations for those working in the current debate." (p. 76)

34. Roski, Stefan, and Rumberg, Antje. 2016. "Simplicity and Economy in Bolzano's Theory of Grounding." *Journal of the History of Philosophy* no. 54:469-496

Abstract: This paper is devoted to Bolzano's theory of grounding (*Abfolge*) in his *Wissenschaftslehre*. Bolzanian grounding is an explanatory consequence relation that is frequently considered an ancestor of the notion of metaphysical grounding. The paper focuses on two principles that concern grounding in the realm of conceptual sciences and relate to traditionally widespread ideas on explanations: the principles, namely, that grounding orders conceptual truths from simple to more complex ones (Simplicity), and that it comes along with a certain theoretical economy among them (Economy). Being spelled out on the basis of Bolzano's notion of deducibility (*Ableitbarkeit*), these principles are revealing for the question to what extent grounding can be considered a formal relation."

35. Roski, Stefan, and Rusnock, Paul. 2014. "Bolzano on Necessary Existence." *Archiv für Geschichte der Philosophie* no. 96:320-359
Abstract: "This paper is devoted to an examination of Bolzano's notion of necessary existence, which has so far received relatively little attention in the literature.
We situate Bolzano's ideas in their historical context and show how he proposed to correct various flaws of his predecessors' definitions. Further, we relate Bolzano's conception to his metaphysical and theological assumptions, arguing that some consequences of his definition which have been deemed counterintuitive by some of his interpreters turn out to be more reasonable given the broadly Leibnizian background of his metaphysics. Finally, we consider some difficulties that arise from Bolzano's evolving views on freedom, which, at least in his early thought, was intimately linked with contingency. In an appendix, we discuss a recent debate on Bolzano's notion of necessary truth between Textor and Rusnock that has some bearing on our overall line of interpretation of Bolzano's notion of *necessary existence*."
References
Rusnock, P. 2012. "On Bolzano's Conception of Necessary Truth". *British Journal of the History of Philosophy* 20, 817-837.
Textor, M. 2013. "Bolzano on the Source of Necessity: A Reply to Rusnock". *British Journal of the History of Philosophy* 21, 381-392.

36. Roski, Stefan, and Schnieder, Benjamin. 2019. "Fundamental Truths and the Principle of Sufficient Reason in Bolzano's Theory of Grounding." *Journal of the History of Philosophy* no. 57:675-706
Abstract: "Bernard Bolzano developed his theory of grounding in opposition to the rationalists' Principle of Sufficient Reason (the PSR). He argued that the PSR fails because there are fundamental, that is, ungrounded truths. The current paper examines Bolzano's views on fundamentality, relating them to ongoing debates about grounding and fundamentality."

37. ———. 2022. "Introdfuction: A Survey of Bolzano's Theory of Grounding." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 4-34. New York: Oxford University Press
"In this survey paper, we pursue three aims:
- First, we briefly sketch the origins of Bolzano's views on grounding and the role that grounding plays in his philosophy.
- Second, we give an overview of Bolzano's mature conception of grounding, focussing on its most detailed exposition, which can be found in his *Theory of Science*.
- Third, we introduce elements and terminology from Bolzano's conceptual framework that are required to understand his theory of grounding." (p. 4)

38. ———, eds. 2022. *Bolzano's Philosophy of Grounding: Translations and Studies*. New York: Oxford University Press
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Mathematics 45; 5. Middle Period: Theology and Metaphysics 85; 6. Mature Period: A Theory of Grounding 107; Part III: Research Papers on Bolzano's Theory; 7. Mark Malink: Aristotle and Bolzano on Grounding 221; 8. Kevin Mulligan: Logic, Logical Norms, and (Normative) Grounding 244, 9. Kit Fine:Some Remarks on Bolzano on Ground 276; 10. Mark Textor: Grounding, Simplicity, and Repetition 301; 11. Francesca Poggiolesi: Bolzano, (the Appropriate) Relevant Logic, and Grounding Rules for Implication 319; 12. Edgar Morscher: The Grounds of Moral 'Truths' 343; 13. Paul Rusnock: Grounding in Practice: Bolzano's *Purely Analytic Proof* in Light of the Contributions 364; 14. Marc Lange: Bolzano, the Parallelogram of Forces, and Scientific Explanation 394; 15. Benjamin Schnieder: A Fundamental Being: Bolzano's Cosmological Argument and Its Leibnizian Roots 418; Glossary of German Terms 445; Name Index 447; Subject Index 450-458.

39. Rumberg, Antje. 2013. "Bolzano's Concept of Grounding (*Abfolge*) Against the Background of Normal Proofs." *Review of Symbolic Logic* no. 6:424-459
Abstract: "In this paper, I provide a thorough discussion and reconstruction of Bernard Bolzano's theory of grounding and a detailed investigation into the parallels between his concept of grounding and current notions of normal proofs. Grounding (*Abfolge*) is an objective ground-consequence relation among true propositions that is explanatory in nature. The grounding relation plays a crucial role in Bolzano's proof-theory, and it is essential for his views on the ideal buildup of scientific theories. Occasionally, similarities have been pointed out between Bolzano's ideas on grounding and cut-free proofs in Gentzen's sequent calculus. My thesis is, however, that they bear an even stronger resemblance to the normal natural deduction proofs employed in proof-theoretic semantics in the tradition of Dummett and Prawitz."

40. Rusnock, Paul. 1997. "Bolzano and the Traditions of Analysis." *Grazer Philosophische Studien* no. 53:61-85
Abstract: "Russell, in his *History of Western Philosophy,* wrote that modern analytical philosophy had its origins in the construction of modern functional analysis by Weierstrass and others. As it turns out, Bolzano, in the first four decades of the nineteenth century, had already made important contributions 'to the creation of "Weierstrassian" analysis, some of which were well known to Weierstrass and his circle. In addition, his mathematical research was guided by a methodology which articulated many of the central principles of modern philosophical analysis. That Russell was able to discover philosophical content within mathematical analysis was thus not surprising, for it had been carefully put there in the first place. Bolzano can and should, accordingly, be viewed as a founder of modern analytical philosophy, and not necessarily as an uninfluential one. This paper considers his work in mathematical and philosophical analysis against some of the relevant historical background."

41. ———. 1997. "Remaking Mathematics: Bolzano reads Lagrange." *Acta Analytica* no. 18:51-72
"With Cauchy, Bolzano was among the most thorough and acute of Lagrange's readers, and it is clear that Bolzano had a good deal of respect for him as a mathematician, going through his treatises pencil in hand as soon as they were published, and occupying himself with many of the same questions. Like Lagrange, Bolzano was dissatisfied with the state of the foundations of analysis; like him he sought to provide an autonomous foundation for this branch of mathematics, one free from appeals to infinitesimals, geometry, and motion. Bolzano also appears to have respected Lagrange's opinion on the contents of analysis.
(...)
This broad agreement on content, however, was accompanied by sharp disagreements concerning method. Indeed, Bolzano chose his early subjects in part precisely in order to accentuate these differences. For Lagrange's entire approach to analysis was out of harmony with Bolzano's philosophy of science. And as Lagrange's work was in many ways the highest expression of analysis around the

beginning of the nineteenth century, Bolzano's criticisms applied quite generally to the state of mathematics at the time. The difficulties which he found were not of the kind that one could hope to resolve by small changes of detail. They were, rather, systemic. What was required, according to Bolzano, was no less than a "complete transformation" of mathematics, at least of those parts which are not to be rejected as completely incorrect.(3) Not one to make such a statement idly, Bolzano had already been working on the task for over a decade, and would spend a good part of the rest of his life attempting to finish the work, rebuilding mathematics from the ground up in line with his methodology. This led to a detailed confrontation with eighteenth-century and notably Lagrangian mathematics; and it is here, in Bolzano's criticisms, and the alternatives he proposes, that we find the unmistakable imprint of his philosophy." (pp. 2-3)
(3) *Rein analytischer Beweis des Lehrsatzes, daß zwischen je zwey Werthe, die eine entgegengesetzes Resultat gewähren, mindestens eine reele Würzel der Gleichung liege* (Prague, 1817), Preface; English translation by S. B. Russ, *Historia Mathematica* 7 (1980) 156-185.

42. ———. 1999. "Philosophy of Mathematics: Bolzano's Responses to Kant and Lagrange." *Revue d'Histoire des Sciences* no. 52 (3-4):399-428
Summary: "Bolzano's philosophy of mathematics is presented through a consideration of his critical responses to Kant and Lagrange."
"In a late essay, Bolzano describes the philosophy of mathematics as an activity aimed at discovering the objective grounds of propositions which we already know with the greatest certainty and evidence (1). For him, philosophy of mathematics was simply what we would now call foundational research in the broadest sense - that is, it was not just a matter of « ultimate » foundations (for instance set theory, logic, or the like), but also of the foundations of particular mathematical theories (for instance geometry, the calculus, combinatorics...). Bolzano was certainly committed to dealing with questions of ultimate foundations, with developing a unified system of mathematics from first principles - his detailed investigations of set theory and logic bear ample witness to this. He also understood, however, that foundational inquiries could be, at least provisionally, local. One could, as he explained in the *Contributions to a better-founded presentation of mathematics* of 1810, assume certain propositions as locally primitive, deferring until a later date their proof from more basic principles (2). No sharp line can be drawn to separate such local questions from those of ultimate foundations. Searching for underlying principles, in whatever domain and at whatever level, was an activity he quite plausibly and in line with tradition regarded as philosophical." (pp. 399-400)
(1) Bernard Bolzano, *Was ist Philosophie?* (Wien, 1849), 23.
(2) Bernard Bolzano, *Beyträge zu einer begründeteren Darstellung der Mathematik* (Prag, 1810), part II, § 11 (hereafter: *Beyträge*).

43. ———. 2000. *Bolzano's Philosophy and the Emergence of Modern Mathematics*. Amsterdam: Rodopi
"In his own time, Bolzano was known primarily for his highly public life as a social and religious reformer, one of the leading figures of the Bohemian Enlightenment. In mathematics and logic - the concerns of this book - Bolzano was no less a reformer, developing strikingly modem views on logic, and attempting to recast mathematics in line with the methods set out in this new logic. He pursued this project doggedly,
attempting to carry it through to the last details. The results, although incomplete, are impressive, and worthy of our attention.
I have tried in this book to give an adequate sketch of Bolzano as a philosopher of mathematics and as a philosophical mathematician.
Within his mathematical work, I have chosen to focus on his research in the foundations of real analysis, as it is here where he had the greatest success, and where the positive imprint of his philosophical views is most apparent. Of his vast writings on logic, I have confined my attention mainly to those parts which bear most directly on mathematical method. Much of Bolzano's mathematics and logic

will no doubt appear quite familiar, and it is easy to forget just how new and strange this territory was when Bolzano - often on his own - first moved into it. For this reason, I have attempted also to convey something of the historical context of his work." (pp. 4-5)

44. ———. 2011. "Kant and Bolzano on Logical Form." *Kant-Studien* no. 102
Abstract: "In the works of Kant and his followers, the notion of form plays an important role in explaining the apriority, necessity and certainty of logic. Bernard Bolzano (1781–1848), an important early critic of Kant, found the Kantians' definitions of form imprecise and their explanations of the special status of logic deeply unsatisfying. Proposing his own conception of form, Bolzano developed radically different views on logic, truth in virtue of form, and other matters. This essay presents Bolzano's views in the light of his criticisms of the Kantian logicians."

45. ———. 2012. "Remarks on Bolzano's Conception of Necessary Truth." *British Journal for the History of Philosophy* no. 20:817-837
Abstract: "This essay presents a new interpretation of Bolzano's account of necessary truth as set out in §182 of the *Theory of Science*. According to this interpretation, Bolzano's conception is closely related to that of Leibniz, with some important differences. In the first place, Bolzano's conception of necessary truth embraces not only what Leibniz called metaphysical or brute necessities but also moral necessities (truths grounded in God's choice of the best among all metaphysical possibilities). Second, in marked contrast to Leibniz, Bolzano maintains that there is still plenty of room for contingency even on this broader conception of necessity."

46. ———. 2013. "On Bolzano's Concept of a Sum." *History and Philosophy of Logic* no. 34:155-169
Abstract: "Alongside his groundbreaking work in logic, Bernard Bolzano (1781–1848) made important contributions to ontology, notably with his theory of collections. Recent work has done much to elucidate Bolzano's conceptions, but his notion of a sum has proved stubbornly resistant to complete understanding. This paper offers a new interpretation of Bolzano's concept of a sum. I argue that, although Bolzano's presentation is defective, his conception is unexceptionable, and has important applications, notably in his work on the foundations of arithmetic."

47. ———. 2013. "Kant and Bolzano on Analyticity." *Archiv für Geschichte der Philosophie* no. 95:298-335
Abstract: "The history of speculation on a notion or notions called analyticity, now usually characterized as truth in virtue of meanings and independently of fact, is often viewed from the perspective of the Quine-Carnap dispute. Previous characterizations, due to Kant, Frege and others, are then seen as being of a piece with Carnap's various definitions of analyticity, and thus open to Quine's objections. Seen from this point of view, Bolzano's claims about analyticity appear downright bizarre: for on his conception, analyticity is not only non-linguistic, but also independent of both apriority and necessity. In this paper, it is argued that the problem lies not with Bolzano, but rather with the received historical account, especially its interpretation of Kant."

48. ———. 2022. "Grounding in Practice: Bolzano's Purely Analytic Proof in Light of the *Contributions*." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 364-393. New York: Oxford University Press
"Bolzano's best-known mathematical work, the *Rein analytischer Beweis* of 1817, promises to deliver a ground-revealing proof of an important theorem from the theory of equations, which Bolzano shows to follow from (a generalization of) the intermediate value theorem. In his paper Paul Rusnock explains and assesses this promise against the background of Bolzano's early account of mathematical method, in which the idea of grounding plays a central role."

49. Rusnock, Paul, and Burke, Mark. 2010. "Etchemendy and Bolzano on Logical Consequence." *History and Philosophy of Logic* no. 31:3-29
Abstract: "In a series of publications beginning in the 1980s, John Etchemendy has argued that the standard semantical account of logical consequence, due in its essentials to Alfred Tarski, is fundamentally mistaken. He argues that, while Tarski's definition requires us to classify the terms of a language as logical or non-logical, no such division is guaranteed to deliver the correct extension of our pre-theoretical or intuitive consequence relation. In addition, and perhaps more importantly, Tarski's account is claimed to be incapable of explaining an essential modal/epistemological feature of consequence, namely, its necessity and apriority. Bernard Bolzano (1781-1848) is widely recognized as having anticipated Tarski's definition in his *Wissenschaftslehre* (or *Theory of Science*) of 1837. Because of the similarities between his account and Tarski's, Etchemendy's arguments have also been extended to cover Bolzano. The purpose of this article is to consider Bolzano's theory in the light of these criticisms. We argue that, due to important differences between Bolzano's and Tarski's theories, Etchemendy's objections do not apply immediately to Bolzano's account of consequence. Moreover, Bolzano's writings contain the elements of a detailed philosophical response to Etchemendy."

50. Rusnock, Paul, and George, Rolf. 2004. "Bolzano as Logician." In *The Rise of Modern Logic: from Leibniz to Frege*, edited by Gabbay, Dov and Woods, Jean, 177-205. Amsterdam: North-Holland
Handbook of the History of Logic. Vol. 3.
"Bernard Bolzano (1781-1848) stands out with Frege as one of the great logicians of the nineteenth century. His approach to logic, set out in the *Theory of Science* [WL] of 1837, marks a fundamental reorientation of the subject on many fronts, one which is as radical as any in the history of the field. In sharp contrast to many of his contemporaries, Bolzano insisted upon a rigorous separation of logic from psychology. It should be possible, he thought, to characterize propositions, ideas, inferences, and the axiomatic organization of sciences without reference to a thinking subject. Consistently pursuing this approach to logic and methodology, Bolzano developed important accounts of formal semantics and formal axiomatics. A talented mathematician, Bolzano developed his logic in conjunction with his mathematical research. Among the first to work on the foundations of mathematics in the modern sense of the term, he made a number of key discoveries in analysis, topology, and set theory, and had a significant influence on the development of mathematics in the nineteenth century. In logic, Bolzano is best remembered for his variation logic (section 4.2 below), a surprisingly subtle and rigorous development of formal semantics. In this article, we discuss Bolzano's logic along with some of his work in the foundations of mathematics which has some bearing on logic." (p. 177)

51. Rusnock, Paul, and Šebestik, Jan. 2013. "The *Beyträge* at 200: Bolzano's Quiet Revolution in the Philosophy of Mathematics." *Journal for the History of Analytical Philosophy* no. 1:1-14
Abstract: "This paper surveys Bolzano's *Beyträge zu einer begründeteren Darstellung der Mathematik* (Contributions to a better-grounded presentation of mathematics) on the 200th anniversary of its publication. The first and only published issue presents a definition of mathematics, a classification of its subdisciplines, and an essay on mathematical method, or logic. Though underdeveloped in some areas (including, somewhat surprisingly, in logic), it is nonetheless a radically innovative work, where Bolzano presents a remarkably modern account of axiomatics and the epistemology of the formal sciences. We also discuss the second, unfinished and unpublished issue, where Bolzano develops his views on universal mathematics. Here we find the beginnings of his theory of collections, for him the most fundamental of the mathematical disciplines. Though not exactly the same as the later Cantorian set theory, Bolzano's theory of collections was used in very similar ways in mathematics, notably in analysis. In

retrospect, Bolzano's debut in philosophy was a remarkably successful one, though its fruits would only become generally known much later."

52. ———. 2019. *Bernard Bolzano: His Life and Work*. New York: Oxford University Press

"Yet interest in Bolzano's theoretical work has rarely extended farther than mere curiosity. Where Frege, for instance, has been the subject of many studies, few English-speaking philosophers have felt moved to look into the details of Bolzano's work. This is more than a pity, since Bolzano did not simply anticipate what others later developed, but has original things to say that are of enduring interest. One of the most remarkable philosophers of the nineteenth century, his works are still very much worth studying today, so solid is their foundation, so meticulous their detail. Quine might have done well, for instance, to have considered what Bolzano had to say about the analytic/synthetic distinction, or about the *a priori*, Putnam and Kripke to what Bolzano had to contribute to their discussions of indexicals and natural kind terms. Frege himself, as Alwin Korselt [*] pointed out in a none-too-friendly exchange over the foundations of geometry, might have learned a few things about logical consequence from him.

(...)

In the English-speaking world, Bolzano is best known for his work in logic and mathematics. There are certainly things of great importance and beauty in these parts of his work. We have already written, each of us, on these matters, and will have more to say about them in this book. But a faithful portrait of Bolzano cannot limit itself to this, for until he was 40 years old, he was only able to pursue these subjects in his spare time. With his considerable gifts in these non-controversial areas, he certainly might have led a distinguished life of speculation as a mathematician or philosopher. Instead he chose quite deliberately to plunge into the turbulent political life of his homeland, applying his formidable intelligence, energy, and determination to the reform of his society and its institutions. It is here that we shall begin." (pp. 2-3)

[*] Korselt, Alwin. "Über die Grundlagen der Geometrie." *Jahresberiehte der Deutschen Mathematikervereinigung*, 12 (1903): 402-407.