

Theory and History of Ontology

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Selected Bibliography on Bernard Bolzano's Contributions to Logic and Ontology. First Part: A - C

STUDIES ON BERNARD BOLZANO'S LOGIC AND ONTOLOGY

Abbreviations: WL = Bolzano's *Wissenschaftslehre* (1837)

Two introductory works on Bolzano's philosophy are available in English:

Edgar Morscher, *Bernard Bolzano's Life and Work*, Sankt Augustin, Academia Verlag, 2008;

Sandra Lapointe, *Bolzano's Theoretical Philosophy. An Introduction*, New York, Palgrave Macmillan, 2011.

1. *Bernard Bolzano, 1781-1848 Bicentenary. Impact of Bolzano's Epoch on the Development of Science.* 1982. *Acta Historiae Rerum Naturalium Necnon Technicarum*. Prague: Institute of Czechoslovak and General History CSAS.

Proceedings of the Bernard Bolzano's Bicentenary Conference, Prague, September 7-12, 1981.

Contents: (only the contributions on Bolzano are cited): Zdeněk Ceska: B. Bolzano and the Charles University 3-8; Lubos Novy: Bolzano's contribution to science and society 9-23; Günter Kröber: Bernard Bolzano und das Problem des wissenschaftlich-technischen und sozialen Fortschrittes 103-128; Jan Janko: Veränderungen der Naturgeschichte und ihre Differenziation in der I. Hälfte des 19. Jahrhunderts 129-154; Joseph W. Dauben: Progress of mathematics in the early 19th century: Context, contents and consequences 223-260; Jan Berg: A requirement for the logical basis of scientific theories implied by Bolzano's logic of variation 415-425; Karel Berka: B. Bolzano's philosophy of science 427-442; Jonathan Cohen: Bolzano's theory of induction 443-457; Helmut Mettler: Bernard Bolzano Beitrag zum Gestaltwandel der Logik 479-489.

2. "Bolzano Studien." 1987. *Philosophia Naturalis* no. 24:351-499.

Content (Essays in English):

II. Basic Questions of Logic and Semantics.

Jan Berg: Bolzano and Situation Semantics: Variations on a Theme of Variation 373-377; Peter Simons: Bolzano, Tarski, and the Limits of Logic 378-405;

III. On the Problem of Paradoxism.

Jan Berg: Is Russell's Antinomy Derivable in Bolzano's Logic? 406-413;

IV: Probability, Induction and Syllogistic.

Jan Berg: Bolzano on Induction 442-446;

V. Contributions on Bolzano's Metaphysics.

Rolf George: Bolzano on Time 452-468.

3. *Bolzano's Wissenschaftslehre 1837-1987. International Workshop (Firenze, 16-19 September 1987).* 1992. Firenze: Leo S. Olschki.

Content: Premessa V; Rolf George: Concepts of Consequence 3; Detlef D. Spalt: Bolzano's Zahlbegriffe. Bislang Übersehene Marksteine Feudal-absolutistischer Mathematik 27; Ettore Casari: An Interpretation of Some Ontological and Semantical Notions in Bolzano's Logic 55; Jan Berg: The Connection Between Bolzano's Logic of Variation and His Theory of Probability 107; Benson Mates: Bolzano and Ancient Pyrrhonism 121; Karel Berka: Bolzanos Lehre vom natürlichen Schliessen 141; Jan Sebestik: The

construction of Bolzano's Logical System 163; Carlo Cellucci: Bolzano and Multiple-Conclusion Logic 179; Rudolf Haller: Bolzano and Austrian Philosophy 191; Massimo Mugnai: Leibniz and Bolzano on the Realm of Truths 207; Bob van Rootselaar: Axiomatics in Bolzano's Logico-Mathematical Research 221-230.

4. "Bolzano and Analytic Philosophy." 1997. *Grazer Philosophische Studien* no. 53:1-266.

Edited by Wolfgang Künne, Mark Siebel, Mark Textor.

Proceedings of the International Symposium held in Hamburg 3rd-t5th January 1997.

Contents: Preface; VI; Dagfin Føllesdal: Bolzano's Legacy 1; Jan Berg: Bolzano, the Prescient Encyclopedist 13; Jan Sebestik: Bolzano, Exner and the Origins of Analytical Philosophy 33; Paul Rusnock: Bolzano and the Traditions of Analysis 61; Peter Simons: Bolzano on Collections 87; Ali Behboud: Remarks on Bolzano's Collections 109; Mark Siebel: Variation, Derivability and Necessity 117; Edgar Morscher: Bolzano's Method of Variation: Three Puzzles 139; Rolf George: Bolzano's Programme and Abstract Objects 167; Mark Textor: Bolzano's Sententialism 181; Wolfgang Künne: Propositions in Bolzano and Frege 203; Michael Dummett: Comments on Wolfgang Künne's Paper 241; Carsten Uwe Gieske: Bolzano's Notion of Testifying 249-266.

5. "Bernard Bolzano." 1999. *Revue d'Histoire des Sciences* no. 52:339-506.

The following contributions are in English:

Charles Chihara: Frege's and Bolzano's rationalist conceptions of arithmetic 343; Joëlle Proust: Bolzano's theory of representation 363; Johannes Hafner: Bolzano's criticism of indirect proofs 385; Paul Rusnock: Philosophy of mathematics: Bolzano's responses to Kant and Lagrange 399.

6. "Bolzano and Kant." 2012. *Grazer Philosophische Studien* no. 85.

Guest Editor: Sandra Lapointe.

Table of Contents. Sandra Lapointe: Introduction 1; Sandra Lapointe: Is Logic Formal? Bolzano, Kant and the Kantian Logicians 11; Nicholas F. Stang: A Kantian Reply to Bolzano's Critique of Kant's Analytic-Synthetic Distinction 33; Clinton Tolley: Bolzano and Kant on the Place of Subjectivity in a *Wissenschaftslehre* 63; Timothy Rosenkoetter: Kant and Bolzano on the Singularity of Intuitions 89; Waldemar Rohloff: From Ordinary Language to Definition in Kant and Bolzano 131-149.

7. Baltzer-Jaray, Kimberly. 2006. "Reinach and Bolzano. Towards a Theory of Pure Logic." *Symposium. Journal of the Canadian Society for Continental Philosophy* no. 10:473-502.

"The work of Adolf Reinach (1883-1917) on states of affairs, judgment, and speech acts bears striking similarities to Bernard Bolzano's work in the area of general logic. It is my belief that these similarities suggest that Reinach used Bolzano's logical work to assist with his own. Three considerations support this view. First, Bolzano's work in *Die Wissenschaftslehre* (Theory of Science) was considered by Husserl to be the necessary foundation for any work in logic. Second, Bolzano's logic was a suitable alternative to Immanuel Kant's in that he formulated his essential relations as inexistent yet real, not Platonic or belonging to a transcendental realm. Third, Reinach did not openly criticize Bolzano in the manner he did the Austrians of the Brentano school, suggesting that Bolzano's logic was more complementary with his own. Due to his untimely death in 1917, Reinach's work on states of affairs and logic remains incomplete, some of it even lost or destroyed. I shall here offer a few brief remarks about Husserl as he was Reinach's mentor and friend, but an in depth discussion of the differences between Reinach and Husserl will not be offered in this paper. Secondary literature tells us that Reinach admired Husserl's *Logical Investigations*, in which phenomenology was said to concern itself with "primarily the discovery of the terra firma of pure logic, of the Sachen (things) in the sense of objective entities in general and of general essences in particular," and further "this phenomenology must bring to pure expression, must describe in terms of their essential concepts and their governing formulae of essence, the essences which directly make themselves known in intuition, and the connections which have their roots purely in such essences." These acts of discovering and describing essences or things themselves became the foundation of Reinach's realist ontology: things themselves surround us in the world and our access to them does not require a transcendental turn. It was precisely this realist foundation that allowed Reinach to develop and extend his phenomenological work to logic, legal philosophy, and speech acts as well. This conception of the nature and goal of phenomenology allowed Reinach and other phenomenologists a manner in which to analyze experience with its essential connections without either falling prey to psychologism or resorting to Platonism: phenomenology for them was truly a realist alternative."

8. Bar-Hillel, Yehoshua. 1950. "Bolzano's Definition of Analytic Propositions." *Methodos*:32-55.

Published also in *Theoria* 16, 1950, pp. 91-117.

Reprinted in: *Aspects of language. Essays and lectures on philosophy of language, linguistic philosophy and methodology of linguistics* - Jerusalem - The Magnes Press - The Hebrew University, 1970 pp. 3-28.

"In view of recent discussions on the nature of analytic truth, it should be rather interesting to inquire into the treatment which this subject received by the most outstanding logician of the first half of the 19th century, the Austrian philosopher, theologian, and physicist Bernard Bolzano.

Our investigation will turn upon section 148 of Bolzano's four volumed masterwork *Wissenschaftslehre* (1837). Only occasionally shall we need to refer to other parts of this work. This section, headed "Analytic or Synthetic Propositions", comprises pages 83-89 of the second volume and is divided into three subsections of less than two pages altogether, followed by four annotations, filling the next five pages. I dwell so long upon these bibliographical particulars only to bring into full light the wealth of systematic and historic material contained in these few pages.

1. Pre-History.

Bolzano's aim, in § 148, was to define a concept which could serve as an adequate explication for what is now commonly termed 'logical truth'. Though this aim is nowhere explicitly stated, there can be no doubt about it, just as Kant before him and many logicians after him doubtless aimed at the same target when they proposed their respective definitions.

Bolzano devotes the greater part of his fourth annotation the discussion of many such attempts made by his predecessors and contemporaries. He mentions *Aristotle*, *Locke*, *Crusius* (the German logician of the first half of the 18th century who was probably the first to use the terms 'analytic' and 'synthetic' in their Kantian senses), Kant and many other minor philosophers. He easily succeeds in proving the inadequateness of Kant's two definitions for 'analytic', the one given in his *Logik* and equating, in effect, Analytic (1) with Identical, the other much better known in the introduction to the Critique of Pure Reason, where he proposes to call propositions 'analytic', whose predicate-concept is contained (perhaps in a hidden manner) in the subject-notion. Bolzano points out (p. 87) the vagueness of the term 'contained' and argues that, according to a quite natural interpretation of this term, the proposition « The father of Alexander, King of Macedonia, was King of Macedonia » ought to be analytic, a consequence which Kant certainly did not intend to be drawn.

But to even more refined versions of Kant's definition, given by some of his followers, replacing the vague 'contained' by more concise terms, such as those making use of 'essential characteristics', Bolzano objects that only one type of proposition conforms to them, namely 'A (which is B) is B'. But should not, continues Bolzano, also propositions of the type 'Every object is either B or non-B' be counted among the analytic propositions?

Having thus convinced himself of the inadequateness of all prior approaches, he started to attack the subject along a new and highly original line." (pp. 3-4 of the reprint).

9. ———. 1952. "Bolzano's Propositional Logic." *Archiv für Mathematische Logik und Grundlagenforschung* no. 1:65-98.

"Bar-Hillel's papers are two of the most important studies on Bolzano's logic" Jan Berg.

Reprinted in: *Aspects of language. Essays and lectures on philosophy of language, linguistic philosophy and methodology of linguistics* - Jerusalem - The Magnes Press - The Hebrew University, 1970. pp. 33-68.

"1848 is a remarkable year not only in general history; in the history of human culture and thought it will be remembered also as the birth year of *G. Frege*, "the greatest logician of the 19th century", (2) and should be remembered as the year in which the death of the greatest logician between Leibniz and Frege, the Czech Bernard Bolzano, occurred. So far, little has been done to evaluate his important contributions to logical theory, (3) and I hope that the present article will help to undo this undeserved wrong.

The purpose of this article is very restricted: only a small part of Bolzano's investigations will be dealt with, i.e. his propositional logic, and even this in a limited degree. This theory is in my opinion not only a master-work of outstanding historical interest, I also believe that it contains many features neglected even by modern symbolic logic and nevertheless worthy of close study. I am convinced that such a study will considerably enrich our logical technique and terminology.

Since our principal aim is to emphasize the impact which Bolzano's ideas should have on contemporary logic, I shall allow myself to depart, sometimes considerably, from his original account and even to disregard parts of his theory unacceptable to us which do not play any decisive role in its construction, all this, of course, after due warning shall have been given.

I shall summarize the contents of §§147, 154-160 of Bolzano's *Wissenschaftslehre* (1837), with which alone this study is concerned, in 28 definitions and 95 theorems. Most of these theorems will not be proved, for the sake of brevity, but the reader will, in general, be able to supplement the proofs by himself. Many definitions and a few theorems will be illustrated by simple examples. Major departures

from Bolzano's original account will be specially mentioned and justified.

In the second part of the study I shall outline the place of Bolzano's contribution within the framework of modern semantics, by its detailed comparison with the corresponding parts of R. Carnap's two volumes of *Studies in Semantics*. This comparison will give us a certain perspective on the bearing of Bolzano's highly original innovations for modern research, and on the other hand enable us to see clearly the precise nature of some of his shortcomings."

(1) This article has been written as an outcome of conversations with Professor Hugo Bergman of the Hebrew University, Jerusalem, and a joint reading of the relevant passages of Bolzano's *Wissenschaftslehre*. It is to Professor Bergman that I owe the general ideas on which this paper is based.

(2) According to A. Tarski, *Introduction to Logic*, 1941, p. 19.

(3) The following is a list of the most important articles dealing mainly with Bolzano's contributions to logic which have appeared in the last two decades:

W. Dubislav Bolzano as Vorlauffer der mathematischen Logik", *Philosophisches Jahrbuch der Görres-Gesellschaft*, vol. 44 (1931), pp. 448-456.

H. Scholz, "Die Wissenschaftslehre Bolzanos", *Semesterberichte*, 9. Semester, 1936/37, pp. 1-53.

H. Scholz, "Die Wissenschaftslehre Bolzanos", *Abhandlungen der Fries'schen Schule*, n. s. vol. 6 (1937), pp. 399-472.

H. R. Smart, "Bolzano's Logic", *The Philosophical Review*, vol. 53 (1944), pp. 513-533.

I have not been able to get hold of Scholz's second article, but since it is, according to the *Journal of Symbolic Logic*, only a somewhat broader version of his first article, the loss is probably not too great. My quotations from Scholz will therefore refer always to his first article.

10. Behboud, Ali. 1997. "Remarks on Bolzano's Collections." *Grazer Philosophische Studien* no. 53:109-115.
11. Benoist, Jocelyn. 2002. "Husserl and Bolzano." In *Phenomenology World-Wide: Foundations, Expanding Dynamisms, Life-Engagements. A Guide for Research and Study*, edited by Tymieniecka, Anna-Teresa, 98-100. Dordrecht: Kluwer.
12. Benthem, Johan van. 1984. *Lessons from Bolzano*. Stanford: Center for the Study of Language and Information, Leland Stanford Junior University.

"Bernard Bolzano's contributions to logic, largely unnoticed in the 19th century, have been receiving ever more attention from modern logicians (cf. Scholz, 1937; Berg, 1962; Corcoran, 1975). As a result, it has already become something of a commonplace to credit Bolzano with the discovery of the notion of logical consequence in the semantic sense. Now, this particular attribution, whether justified or not, would at best establish a historical link between modern logical concerns and Bolzano's work. The purpose of the present note, however, is to bring out three important aspects of that work that are still of contemporary systematic interest. No detailed textual study of Bolzano is needed to substantiate our suggestions. We shall refer to well-documented 'public' aspects of the 'Wissenschaftslehre' (Bolzano, 1837), pointing out their more general logical significance." (p. 1).

13. ———. 1985. "The Variety of Consequence, According to Bolzano." *Studia Logica* no. 44 (4):389-403.

"Contemporary historians of logic tend to credit Bernard Bolzano with the invention of the semantic notion of consequence, a full century before Tarski. Nevertheless, Bolzano's work played no significant role in the genesis of modern logical semantics. The purpose of this paper is to point out three highly original, and still quite relevant themes in Bolzano's work, being a systematic study of possible types of inference, of consistency, as well as their meta-theory. There are certain analogies with Tarski's concerns here, although the main thrust seems to be different, both philosophically and technically. Thus, if only obliquely, we also provide some additional historical perspective on Tarski's achievement."
14. ———. 2003. "Is There Still Logic in Bolzano's Key?" In *Bernard Bolzanos Leistungen in Logik, Mathematik Und Physik*, edited by Morscher, Edgar, 11-34. Sankt Augustin: Academia Verlag.

"Bolzano is widely seen as the philosopher of abstract propositions, far removed from psychological blemishes. Nevertheless, many themes in this paper suggest links with the actual reasoning performed by non-Platonic humans like us. We saw this with attention to diverse styles of task-dependent reasoning, with degrees of logicity for the expressions of natural language that we actually use, with inferences transferring information across discourse situations, with global architecture of reasoning styles, or with mixtures of such neatly compartmentalized logical activities as semantic evaluation and proof. When we

take all this seriously, it becomes hard not to go one step further, and do something which Frege has forbidden - but probably also Bolzano: take the psychological facts seriously. All the above topics border on cognitive science and the experimental study of human reasoning, and the eventual agenda of modern logic will also have to come to better terms with that than the by now pretty stale slogan of 'anti-psychologism'.

Conclusions.

We have surveyed some aspects of Bolzano's logic from a modern standpoint, stressing in particular his different styles of consequence, the essential ternary nature of consequence when language is taken into account, and the mixed notion of consequence in a model. In all three cases we included some new technical observations to show that the issues are still alive. But the more general thrust is this.

Bolzano's work remains interesting for logic today, both in its general sweep, and in some of its details. Partly, it is attractive precisely because it is so non-mainstream, and hence valuable for modern agenda discussions. Its themes crossing logic and philosophy of science reflect current rapprochements, while its thrust also seems to fit with some themes from AI. Classical mathematical logic has had an Austrian icon in Kurt Gödel: modern logic might consider at least having a Czech-Austrian patron saint." (pp. 30-31).

15. ———. 2013. "Bernard Bolzano's *Wissenschaftslehre*." *Topoi* no. 32:301-303.

16. Berg, Jan. 1962. *Bolzano's Logic*. Stockholm: Almqvist & Wiksell.

Contents: Preface 5; Introduction 7; Abbreviations 12; I. Bolzano's life and work 13; II. A logical frame 33; III. Bolzano's fundamental notions 41; IV. Bolzano's logic of variation 92; V. Other logical theories 146; VI. Bolzano's logic of entailment 151; VII. Bolzano's philosophy of mathematics 165; Bibliography 179; Index of proper names 213-214.

"Bernard Bolzano made essential contributions to, *inter alia*, theology, logic, and mathematics. For political reasons, however, he was prevented from influencing to a full extent the age in which he lived. As a mathematician his name has survived, although many of his most remarkable results were not published until a century after their conception. As a logician he has begun to appear again in scattered articles and comments. In this study I have tried to give an exposition and evaluation of the main ideas of his logic from a modern viewpoint.

Important parts of Bolzano's theories of logic and semantics were new with him, and when these ideas reappear later they were independent of him. This position, in part outside of the historical development, makes it highly pertinent to compare Bolzano's theories directly with modern logic. When tracing the lineage of some of Bolzano's ideas I have even projected the earlier theories onto a modern scheme of reference.

In the first chapter I shall briefly mention the genesis of Bolzano's main works. But I am not primarily interested in the genetic aspect of Bolzano's theories. Therefore, as often as possible I shall consider Bolzano's various formulations of his ideas as if they were parts of a simultaneous whole.

My analysis of Bolzano's achievements in logic, semantics, and mathematical philosophy is based on his mature production after 1820, when he started writing his *magnum opus*, the *Wissenschaftslehre*. The works chiefly consulted are:

(1) *Wissenschaftslehre*, I-IV (Bolzano (15) mostly abbreviated "WL"). [Names followed by parenthesized numerals refer to the bibliography; see the last paragraph of this introduction.]

(2) *Der Briefwechsel B. Bolzano's mit F. Exner* (Bolzano (54), "BE").

(3) *Erste Begriffe der allgemeinen Grössenlehre* (Bolzano (78), "AG").

(4) *Reine Zahlenlehre. Erster Abschnitt. Von dem Begriffe, den allgemeinsten Beschaffenheiten und der Bezeichnungsart der Zahlen* (Bolzano (79), "RZ").

(5) *Unendliche Grössenbegriffe* (Bolzano (81), "UG").

(6) *Bolzano's Wissenschaftslehre und Religionswissenschaft in einer beurtheilenden Uebersicht* (Bolzano (z5), "WU").

(7) *Einleitung zur Grössenlehre* (Bolzano (83), "EG").

(8) *Paradoxien des Unendlichen* (Bolzano (45), "PU").

For (3)-(5) and (7) I have utilized Bolzano's unpublished manuscripts. In general I have presupposed that some editions of (1), (2), and (8) are accessible to the reader.

Chapter I presents in a concentrated form some biographical and bibliographical facts about Bolzano. The manuscripts used are described and dated. Some topics lying outside of logic proper and dealt with in published or unpublished works of Bolzano are also touched upon. The most representative portraits of Bolzano - one of which has not been published before - are reproduced.

Chapter II describes the logical machinery to be used in the formalization and the comparative analysis of Bolzano's logic. This chapter may be read cursorily and used as future occasion may require. The system expounds the so-called elementary logic, i.e., classical predicate logic of first order with identity. In view of its simplicity and non-controversial character, elementary logic seems at present to be an expedient object for comparison in research in the history of logic. To be sure, we know nothing for certain about

the future of logic; maybe the logics of tomorrow will differ greatly from those of today. However, it is likely that our elementary logic will be translatable into or representable within these conceivable new systems. But there is always the possibility that new systems of logic will promote a deeper understanding of certain features of the objects analysed.

Chapters III-VII expound those aspects of Bolzano's theories of logic, semantics, and mathematical philosophy which seem to me fundamental. Certain sections of these chapters are subdivided into two parts; part A describes Bolzano's ideas and part B offers commentaries on A. In order to shorten the exposition, definitions and arguments are sometimes formalized even in A, and sometimes auxiliary notions not found in Bolzano are introduced with explicit caution. In doing so I have always attempted to stay within or very close to Bolzano's sphere of ideas. This does not mean, of course, that I have always followed the order in which Bolzano presents his definitions and theories in the *Wissenschaftslehre* or elsewhere. Nor have I stinted myself at times in giving very free paraphrases of Bolzano's mode of expression. In proving theorems I try to reproduce Bolzano's line of thought without copying his manner of speaking. In part B, Bolzano's logic is compared with modern theories. I attempt to show how certain gaps could be filled in and how Bolzano's theories could be elaborated and made more precise. Moreover, under B some forerunners of Bolzano and selected parts of the modern literature concerning him are discussed.

In the annotated bibliography, part A embraces the literature on Bolzano and his own works. References to part A are given by names followed by numerals within parenthesis. References to part B of the bibliography are effected by placing a "B" after the parenthesized numerals. In references to manuscripts, folio numbers are qualified by "r" and "v", meaning, as usual, recto and verso respectively." (*Introduction*).

17. ———. 1966. "Bolzano's Notion of Proposition." In *Ost Und West in Der Geschichte Des Denkens Und Der Kulturellen Beziehungen. Festschrift Für Eduard Winter Zum 70. Geburtstag. Mit Einem Geleitwort Von A.P. Juskevic*, edited by Steinitz, Wolfgang, 519-526. Berlin: Akademie-Verlag.
18. ———. 1972. "Bolzano's Theory of an Ideal Language." In *Contemporary Philosophy in Scandinavia*, edited by Paul, Anthony, Olson, Raymond and Wright, Georg Henrik von, 405-415. Baltimore, London: John Hopkins Press.

"In his logical inquiries Bolzano employed a partly formalized language embracing an ordinary language extended by constants, variables, and certain technical expressions. In the second volume of the *Wissenschaftslehre* he investigated the relations of this semiformalized philosophical language to colloquial language (WL, sections 127-46, 169-84). He believed that all sentences of colloquial language were 'reducible' to sentences of certain canonical forms expressed in the philosophical language. These canonical sentences were said to mirror their corresponding propositions in the sharpest way.

Had Bolzano's theory of reduction been completely developed it might have resulted in the construction of an ideal language for philosophical analysis. In this ideal language, however, sentences of canonical form would not play quite the same role as the atomic sentence forms on the basis of which more complex forms are built up in modern quantification theory. It seems, on the contrary, that Bolzano intended even the most complicated sentences to have canonical forms or to be reducible to sentences having such form.

This paper attempts a reconstruction of an extensional Bolzanian ideal language on the level of elementary logic. After some preliminary explanations of fundamental notions in Bolzano's logic, the main points of his theory of reduction of sentences are described. Two principles that determine the construction of an elementary Bolzanian ideal language emerge from the exposition. We then move toward building such a language and begin by modifying the standard representation of elementary logic, replacing the universal and existential quantifiers by Hilbert's e-operator. By further modifications of both the syntax and the underlying semantics, a logical language satisfying the two principles is obtained." pp. 405-406.

19. ———. 1977. "Bolzano's Contribution to Logic and Philosophy of Mathematics." In *Logic Colloquium '76*, edited by Gandy, Robin O. and Hyland, John Martin, 147-171. Amsterdam: North-Holland.

"The WL [*Wissenschaftslehre*] was intended merely as a prelude to Bolzano's work on mathematics. His main ambition was to recreate the whole body of contemporary mathematics in accordance with the vision of an abstract hierarchy of true propositions. For Bolzano this task implied the creation of entirely new foundations for certain branches of mathematics, as may be seen from his highly interesting efforts directed toward basing geometry on topological concepts.

In carrying out this program, most of the means of expression of modern quantification theory were in essence available to Bolzano.

He came very close to modern notions of satisfaction, logical truth, consistency and logical consequence. On the other hand, the formal deductive machinery of quantification theory is practically non-existent in Bolzano's works. This syntactic machinery appears only in Frege, who created the first strictly logistical system at the end of the 19th century. Bolzano's lack of interest in developing particular logical calculi most probably stems from his aspects of logic and mathematics and of science in general.

The notion of calculus in the modern logistical sense was first clearly considered by Leibniz. His basic dream was of an effectively decidable, interpreted calculus embracing all "eternal" truth. Bolzano was justifiably critical of this overambitious program and presented instead his own theory of the *Abfolge* structure of nonlinguistic propositions, thereby taking his stand away from that line of development in logic which leads to modern syntactic concept formation. A reason for Bolzano's general lack of interest in questions of logical syntax was no doubt his profoundly intensional, non-linguistic approach to logic. Bolzano's central thesis, that there are abstract objects which differ from both mental occurrences and all kinds of linguistic expressions, has been advocated by later philosophers of the German-speaking countries, inter alia by Lotze, Brentano in his earlier period, Meinong and Frege. Lotze and Frege never refer to Bolzano's work, though, and the others protested their independence of Bolzano. Husserl admits that he received vital influences from Bolzano, but his notions of "ideal" objects derive from Lotze's and not from Bolzano's logic.

Among the great Western philosophers Bolzano is perhaps the least influential. In epistemology, logic and mathematics his most fervent disciples were not able to propagate his ideas with sufficient vigor.

His keen criticism of German idealistic philosophy and his important discoveries in logic: semantics and mathematical philosophy silently died away.

A contributing cause of Bolzano's lack of influence on the development of the philosophical disciplines was, of course, the fact that most of his works were, for political reasons, published anonymously in editions not easily accessible. Furthermore, an immense number of unpublished manuscripts in a partly almost indecipherable handwriting is to be found in archives in Prague and Vienna. Several unfruitful attempts have been made in the last 150 years to bring out more or less complete editions of Bolzano's works. It is to be hoped that the latest venture launched in Stuttgart, West Germany, will prove more successful." (pp. 170-171)

20. ———. 1982. "A Requirement for the Logical Basis of Scientific Theories Implied by Bolzano's Logic of Variation." *Acta Historiae Rerum Naturalium Necnon Technicarum* no. 12:415-425.

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21. ———. 1986. "A Logic of Terms with an Existence Operator." In *Logic and Abstraction. Essays Dedicated to Per Lindström on His Fiftieth Birthday*, edited by Furberg, Mats, Wetterström, Thomas and Aberg, Claes, 71-94. Göteborg: Acta Universitatis Gothoburgensis.
22. ———. 1987. "Bolzano and Situation Semantics: Variation on a Theme of Variation." *Philosophia Naturalis* no. 24:373-377.

"The distinction between logical and non-logical notions plays a significant role in Bolzano's theory and he is fully aware of its importance even though he has to admit that various scholars may differ in their opinion on what a *logical idea* is (WL § 148.3). Tarski has tried to clarify the distinction between logical and non-logical notions (Adam Tarski, *What are logical notions? History and Philosophy of Logic*, 7, 1986, pp. 143-154). A notion of Euclidean geometry, e.g., is invariant under all similarity transformations, and a topological notion is invariant under all continuous transformations. Analogously, a *logical notion* may be conceived of as a concept which is invariant under all bijective mappings of the domain of individuals onto itself. (In this sense even the classical reduction problem of critical realism can be solved: A physical notion is a concept invariant under a Galilei or a Lorentz transformation.) It is possible to vary not only the non-logical ideas-as-such contained in propositions but even some or all logical ideas. Actually, such a variation is implied by the algebraic approach to logic. The propositions of Bolzano correspond to the values of the propositional variables of modern logic.

(...)

Bolzano seems to have intended, however, variation exclusively over non-logical ideas-as-such. That he did not allow a variation of the copula is abundantly clear from his way of introducing the notion of variation in his *Einleitung zur Grossenlehre* (Bolzano (2A7), p. 62). Here he presupposes that only the subject and predicate ideas of a proposition or parts thereof be varied. Furthermore, his proofs of certain variation-logical theorems show that he would not allow a variation of the logical constants of negation, truth, and the copula in the form of an inclusion between ideas-as-such (WL §§ 154.19, 155.21)."

23. ———. 1987. "Is Russell's Antinomy Derivable in Bolzano's Logic?" *Philosophia Naturalis*:406-413.

"In his encyclopedic work *Wissenschaftslehre* Bernard Bolzano expounded a theory of logical truth which constitutes an outstanding achievement in the history of Western thought. This informal theory is essentially based on a substitutional truth-value semantics without certain existence presuppositions and contains a general proof theory. In his substitutional semantics Bolzano introduced notions such as universal validity, consistency, consequence, analyticity, and probability by means of the technique of variation of concepts. In his proof theory he treated the notion of entailment, which is a generalization of a special case of the relation of logical consequence, and studied proof trees generated by the relation of entailment which exhibit the objective connection between all true propositions. In view of this wealth of important notions it seems worthwhile to investigate the possibility of a consistent reconstruction of Bolzano's logic. In particular, it must be examined how his theory fares with a fundamental set-theoretic antinomy such as that of Russell." (p. 406)

"There is, therefore, no such thing as an idea of all ideas which are not objects of themselves, and Bolzano could scarcely be blamed for having no idea of something which does not exist in any sense at all. Under a reasonable interpretation of Bolzano's theory of ideas-as-such there is no trouble-maker around who could generate an antinomy analogous to that of Russell." (p. 411)

24. ———. 1987. "Bolzano on Induction." *Philosophia Naturalis* no. 24:442-446.

"Bolzano combined the fundamental notions of his theory of probability and his proof theory to achieve a logical analysis of the principles of induction. The relation between the conclusion and the premisses of an inference of incomplete induction or analogy is an interior probability relation in Bolzano's sense. The principles of induction endow the relation between the premisses and the conclusion with the character of a relation between ground and consequence. From Bolzano's subsumption of the rule of incomplete induction under the syllogistic rule of Barbara, it follows that this relation is a special case of Bolzanian derivability."

25. ———. 1992. *Ontology without Ultrafilters and Possible Worlds. An Examination of Bolzano's Ontology, Beiträge Zur Bolzano-Forschung; Vol. 1: Sankt Augustin - Academia Verlag.*

Very important, but also very technical

26. ———. 1992. "The Connection between Bolzano's Logic of Variation and His Theory of Probability." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 107-120. Firenze: Leo S. Olschki.

27. ———. 1994. "The Ontological Foundations of Bolzano's Philosophy of Mathematics." In *Logic and Philosophy of Science in Uppsala*, edited by Prawitz, Dag and Westerståhl, Dag, 265-271. Dordrecht: Kluwer.

28. ———. 1997. "Bolzano, the Prescient Encyclopedist." *Grazer Philosophische Studien* no. 53:13-32.

"In his *Wissenschaftslehre* Bernard Bolzano tried to lay down a logically satisfactory foundation of mathematics and theory of probability. Thereby he became aware of the distinction between the actual thoughts and judgments of human beings, their linguistic expressions and the abstract propositions (*Sätze an sich*) and their components (*Vorstellungen an sich*). This ontological distinction is fundamental in Bolzano's thinking paired with a universal world view in the sense that philosophy, mathematics, physics and metaphysics should be build upon the same logical foundations. Bolzano's enterprise is sketched in the light of examples from his logical semantics, proof theory, number theory, theory of truth and his variation logic."

29. ———. 2000. "From Bolzano's Point of View." *The Monist. An International Quarterly Journal of General Philosophical Inquiry* no. 83 (1):47-67.

"In the *Wissenschaftslehre* (1837) Bolzano laid down a logical foundation of mathematics and the theory of probability. By means of the technique of variation of non-logical components in propositions he defined universal validity, logical derivability, and inductive probability. Characteristic features of situation semantics can be represented in the logic of variation. In his proof theory he stated counterparts of Gentzen's cut rule and *Hauptsatz*.

In mathematical manuscripts he defined natural numbers as properties of bijective sets and real numbers

as infinite sequences of rational numbers and accomplished a pioneering specification of infinitesimals. Infinite sets were identified as sets bijective to a proper subset of themselves. In a topological foundation of geometry he defined the notions of a closed, a simple closed, and a connected curve by means of spherical neighborhoods and stated a special case of Jordan's curve theorem. He also defined a notion of continuum and formulated steps toward a recursive definition of spaces with a homogeneous dimension number."

30. ———. 2003. "Bolzano's Heuristics." In *Bernard Bolzanos Leistungen in Logik, Mathematik Und Physik*, edited by Morscher, Edgar, 35-56. Sankt Augustin: Academia Verlag.

"In the fourth part of the *Wissenschaftslehre* [WL], contained in the third volume of the original 1837 edition, Bolzano treats *heuristics* or the "art of discovery", i.e., the "rules to be observed in the search for new truths" (§ 9. Note 3; cf. also § 15.2). The first main section of Bolzano's heuristics embraces the *general rules* of this discipline (§§ 325 -348).

Logic in Bolzano's sense is a theory of science the objects of which are the different sciences and their linguistic representations (§ 15). According to Bolzano a science is a set of true propositions (*Sätze an sich*) worthy of representation in a textbook. Logic or the *theory of science* is a set of rules which are necessary and sufficient for a representation to satisfy certain criteria concerning scientific textbooks (§ 1). In view of this very broad conception of logic it is fairly obvious that heuristics is an integrant part thereof." (p. 35)

This paper was already presented in 1991 at the International Bolzano Symposium in Salzburg, but has never been published since.

31. ———. 2003. "The Importance of Being Bolzano." In *Bernard Bolzanos Leistungen in Logik, Mathematik Und Physik*, edited by Morscher, Edgar, 153-166. Sankt Augustin: Academia Verlag.

"1. Logical consequence

Ever since Aristotle philosophers have occupied themselves with the question whether a given statement follows from another statement. The first published precision of this notion in modern times was undertaken by the Polish logician Alfred Tarski in 1936. Accordingly, a closed formula F is a logical consequence of a set of formulas F if and only if F is true under every interpretation of the nonlogical constants under which all elements of F are true. Logical constants are inter alia connectives of sentential logic (expressed by words like "not", "and", "or", "if - then") and quantifiers of predicate logic (such as "for all" and "there is"); hence, the interpretation of these constants is determined.

But who conceived this notion of logical consequence (*mutatis mutandis*) already a hundred years earlier?"

Right: The Bohemian philosopher, ontologist, logician, mathematician and theologian Bernard Bolzano! Upon substitution of abstract nonlinguistic propositions for closed formulas and variants of propositions for interpretations, we get precisely a special case of Tarski's notion of logical consequence. (A variant of a proposition P is a proposition identical with P up to at least one nonlogical component.) Incidentally, at the university of Warsaw Tarski was a student of Lukasiewicz's who lectured inter alia on Bolzano's logic. Just like Bolzano Tarski admitted being unable to exactly distinguish between logical and nonlogical constants. Not until thirty years later did he formulate a necessary condition for the property of being a logical constant. Furthermore, if all constants of the formal language in question were regarded as logical, the notion of material implication, would emerge. Even this weakest of all notions of consequence was introduced by Bolzano and is playing an important role in some of his deduction rules.

Tarski presupposed a fixed domain as a realm of reference for the interpretations. Even Bolzano did not conceive of a combined quantification over domains and components of propositions. (By introducing a predicate for domains and letting the quantifiers refer to this predicate, however, one can represent all theorems of the model theory developed later on.)

Nowadays we know, of course, that Tarski's notion of logical consequence is unsuitable if the set-theoretic language is enlarged by a generalized existence quantifier expressing that there is an absolutely infinite class C (in the sense that C does not include exactly K elements for any cardinal number K). This esoteric fact of modern set theory cannot, however, diminish our appreciation of Bolzano's achievement.

2. Analytic propositions

A fundamental distinction in Kant's Critique of Pure Reason is that between analytic and synthetic judgements. In modern logical semantics analyticity is often considered a relation between a sentence S , a set of definitions, and a language L . For instance, one can say that S in L is analytic with respect to D if S is a logical consequence of D in L which embraces S and the elements of D .

But who formulated an analogous explication of analyticity within the system of abstract propositions already in the 1830s?

Right: Bernard Bolzano!" (pp. 153-154)

(...)

"6. Situation semantics

In modern so-called situation semantics, established at the beginning of the 1980s by the American logician and linguist Jon Barwise, a notion of consequence is introduced which is stronger than that of Tarski. In situation semantics certain set-theoretic structures are considered models and a situation is a partial submodel thereof. The primitive notion is the confirmation of a sentence in a model by a situation. For example, a sentence of the form of "A or not A" is a logical consequence of any sentence in the sense of Tarski but not a strong consequence of it.

But who discovered this notion of strong consequence even a hundred years earlier?

Right: Bernard Bolzano!

In his logic Bolzano considered not only the variants with respect to the sequence of all nonlogical components of propositions but also the variants with respect to all subsequences. By that counterparts of main laws of situation semantics turn into theorems of Bolzano's logic." (p. 156)

(...)

17. Estimation

Thus some outstanding achievements of Bolzano's on the fields of logic, semantics, and mathematics have been delineated. The fact that the connection of most of these achievements with modern research remained unknown until the 1960s is due to the circumstance that the study of Bolzano's work took a new turn then and that eventually editions of the often hardly legible manuscripts of the literary remains could be published in the Collected Works of Bernard Bolzano.

Moreover, particularly in Bolzano's logical semantics there are many original ideas which have no precise affinity with modern theories. In addition to that he accomplished extensive investigations into concepts of epistemology, philosophy of nature, physics, metaphysics, ethics, and theology." (p. 165)

32. Berka, Karel. 1982. "Bolzano's Philosophy of Science." In *Bernard Bolzano, 1781-1848 Bicentenary. Impact of Bolzano's Epoch on the Development of Science*, 427-442. Prague: Institute of Czechoslovak and General History CSAS.
33. ———. 1983. "The Ideal of Mathematization in B. Bolzano." In *Nature Mathematized. Historical and Philosophical Case Studies in Classical Modern Natural Philosophy*, edited by Shea, William R., 291-298. Reidel: Kluwer.
34. ———. 1988. "Natural Deduction in Bolzano's *Wissenschaftslehre*." In *Intensional Logic, History of Philosophy and Methodology. To Imre Ruzsa on the Occasion of His 65th Birthday*, edited by Bodnár, István M., Maté, András and László, Pólos, 203-212. Budapest: Department of Symbolic Logic, Eotvos University.
35. ———. 1998. "Bernard Bolzano. A Historian of Logic." *History of Science and Technology* no. 31:121-130.
36. Betti, Arianna. 1998. "De Veritate: Another Chapter. The Bolzano-Lesniewski Connection." In *The Lvov-Warsaw School and Contemporary Philosophy*, edited by Kijania-Placek, Katarzyna and Wolenski, Jan, 115-137. Dordrecht: Kluwer.
37. ———. 2006. "Sempiternal Truth. The Bolzano-Twardowski-Lesniewski Axis." In *The Lvov-Warsaw School. The New Generation*, edited by Jadacki, Jacek Jusliuz and Pasniczek, Jacek, 371-399. Amsterdam: Rodopi.
38. ———. 2006. "The Strange Case of Savonarola and the Painted Fish. On the Bolzaniization of Polish Thought." In *Actions, Products, and Things. Brentano and Polish Philosophy*, edited by Chrudzinski, Arkadiusz, 55-81. Frankfurt: Ontos Verlag.

"I have previously discussed in several papers specific Bolzanian elements present in the Polish tradition. This paper will not, for the most part, add anything in particular to that. The new - and rather blunt hypothesis to be put forward here is that, despite appearances, Twardowski also contributed *de facto* to slowing down the reception of Bolzano's most modern logical discoveries. For in Poland Bolzano was to remain one logician among many for rather long. It was chiefly thanks to two factors that Bolzano's star could, slowly, begin to rise in Poland, or, at least, that the fundamental achievements of his logic could be known. One factor is antipsychologistic (more precisely Platonistic) influence coming from Husserl and

from Twardowski's student Lukasiewicz. The other factor is the change in the conception of logic which took Polish logic from, say, Sigwart, to Tarski through Lesniewski and Lukasiewicz," p. 55

39. ———. 2010. "Explanation in Metaphysics and Bolzano's Theory of Ground and Consequence." *Logique et Analyse* no. 56:281-316.

"In "Troubles with Truth-making: Necessitation and Projection." *Erkenntnis* 64: 61-74 (2006a, and in "Truth-Making without Truth-Makers." *Synthese* 152: 21-46 (2006b), Benjamin Schnieder criticizes truthmaking as a relation between entities in the world and the truths those entities 'make true'. In (2006b), his criticism exploits a notion of conceptual explanation that is very similar to Bolzano's grounding. In the first part of this paper, I offer an analysis of Bolzano's grounding. I discuss some open problems and argue that Bolzano's grounding is not a systematization of the ordinary notion of 'because' as others have maintained, but of the technical notion of explanatory proof in the context of an axiomatic conception of (proper) science. On the basis of this analysis, in the second part, I offer a critical discussion of Schnieder 2006b's arguments against truthmaking. I conclude that the latter are not very effective from a methodological point of view and that Bolzano's original position fares better in this respect; still, truthmaker theorists will be able to defend truthmaking only at a high price."

40. ———. 2012. "Bolzano's Universe. Metaphysics, Logic, and Truth." In *Categories of Being. Essays on Metaphysics and Logic*, edited by Haaparanta, Leila and Koskinen, Heikki J., 167-208. New York: Oxford University Press.

"This chapter has two aims. The first aim is to present an overview of Bolzano's universe from the point of view of his metaphysics and its relationship to logic, relying fundamentally on Bolzano's *Wissenschaftslehre*. The author's preferred reading of Bolzano is one according to which he is a 'platonistic nominalist': a platonist about propositions and a nominalist about properties. Bolzano's nominalistic tendencies are particularly conspicuous in his mereological analyses, which play a major role in every aspect of his philosophy. The author's second aim is to answer the open question of whether in Bolzano there is any 'ontology of truth'. This chapter argues for a negative answer. Bolzano does not make room for any special object which plays the role of the counterpart of a truth-bearer, or at least his propositions do not connect semantically in a direct way to anything that would count as such a special object."

41. Beyer, Christian. 2004. "Bolzano and Husserl on Singular Existential Statements." In *Phenomenology and Analysis. Essays on Central European Philosophy*, edited by Chrudzimski, Arkadiusz and Huemer, Wolfgang, 69-88. Frankfurt: Ontos Verlag.

"Which form does the propositional content take that is judged when a given speaker sincerely utters a sentence in order to assert a singular existential statement? Two thought-provoking answers to this question have been proposed by Bernard Bolzano and, when commenting upon Bolzano's proposal, by Edmund Husserl. In Section 1 of this paper the author clarifies what he means by "singular existential statements". In Section 2 Bolzano's proposed analysis is sketched. In Section 3 the author exposes the earlier Husserl's conception of "logical reflection" and draws upon it to explain why Husserl, around 1900, subscribed to Bolzano's proposal. Following this, he reconstructs and considers in detail the later Husserl's discussion of that proposal and Husserl's own mature theory of singular existential statements as manifested in a 1917/18 lecture series, both of which shed light upon a conception that is of central importance for Husserlian phenomenology: the conception of "noematic sense" (Section 4)."

42. Bodnar, Joanne. 1976. *Bolzano and Husserl. Logic and Phenomenology*.

Unpublished Ph.D. thesis, State University of New York at Buffalo, available at ProQuest Dissertation Express.

Contents: Introduction 1; I. Bolzano's Anti-Psychologism 5; II. Bolzano's Theory of Meaning 41; III. Bolzano's Basic Logical Relations 51; IV. Truth to Bolzano 63; V. Husserl's Anti-Psychologism 76; VI. Husserl's Theory of Meaning 99; VII. Basic Logical Relations in Husserl 113; VIII. Truth to Husserl 128; IX. Recapitulation 144; X. Conclusion 151; Bibliography 164-168.

"Bernard Bolzano and Edmund Husserl both present some form of ontological framework for logic rather than a linguistic framework. Their works predate the pragmatic and semantic theories of Tarski and Carnap. Bolzano's *Wissenschaftslehre* appeared in 1837. (2)And Husserl's major logical thinking was formulated before 1935. (3) But neither of them seem receptive to a semantic foundation for logic, because of their rationalist-platonist leanings. Both strongly oppose the view that logic is taken from psychological experience by generalization. They have a viewpoint which is perhaps closer to the

classical outlook than to either of the others, since they consider the foundation of logic to be the acceptance of meanings as entities — entities which are in some important ways related to actual and possible being and its structure.

Bolzano and Husserl each make crucial modifications on traditional platonism as a philosophy of logic. A basic thesis which they both do accept is that the logical entities such as the proposition with its elements and its relations are ideal unities, which are independent of their being thought. But the logical entities are not platonic forms in which spatio-temporal existences "participate" — nor are they determined by spatio-temporal existence in any way. They are independent of the subject or knower as well as of the facts of material existence.(4)

Thus the logical entities are what they are whether they ever come to expression or not. They have a character similar to that of numbers or other "abstract" mathematical objects, but it cannot be said that they arise in experience as abstractions from the empirical world. Although there is disagreement about the question of abstraction in Bolzano — with some Bolzano commentators such as Rolf George seeing little difference between Bolzano and Carnap — this tendency to read Bolzano's work as if he were a pragmatist obscures the originality of Bolzano.(5)

His differences from semantic and empiricist thinking are well worth investigating. Husserl's approach too deserves consideration for its uniqueness. Crediting Bolzano with giving a starting point in philosophy of logic, Husserl "discovered" Bolzano and brought his work out of obscurity. He makes use of the work of Bolzano however, only to transform it thoroughly. If certain common themes are selected for exposition, the positions of Bolzano and Husserl are both seen to be modifications of the classical platonism. This provides a basis for a comparison of Bolzano and Husserl." (pp. 3-5).

(2) Bolzano's work under consideration is *Theory of Science*, ed. and trans. by R. George (Berkeley, 1972), hereafter cited simply as Bolzano. English paginations are used, but section numbers apply to all German editions as well. The *Theory of Science* is a condensation of *Wissenschaftslehre*, vols. 1-4 (Sulzbach, 1837). R. George follows in large measure the F. Kambartel edition of Bolzano's *Wissenschaftslehre* vols. 1-2, entitled *Grundlegung der Logik* (Hamburg, 1963) in which Bolzano's original has been condensed, with the omitted passages summarized by the editor.

(3) Works by Husserl principally under consideration are *Logische Untersuchungen* (1900 and 1913), trans. by J. N. Findlay (New York, 1976); and *Formale und Transzendente Logik* (1929), trans. by D. Cairns (Hague, 1969).

(4) See: U. Neemann, *Bernard Bolzanos Lehre von Anschauung und Begriff in ihrer Bedeutung für erkenntnistheoretische und pädagogische Probleme* (Paderborn, 1972) pp. 81 and 144. for discussions of how Bolzano's logical entities differ from Plato's forms and from Kant's subjective categories.

(5) See: R. George, "Editor's Introduction" in Bolzano's *Theory of Science*, (Berkeley, 1972) p. xxx. Also note J. Berg, *Bolzano's Logic* (Stockholm, 1962) pp. 49-50, where he expresses the view that Bolzano takes logical entities as abstractions.

43. Cantù, Paola. 2011. "Bolzano Versus Kant: Mathematics as a Scientia Universalis." In *Philosophical Papers Dedicated to Kevin Mulligan*, edited by Reboul, Anne. Genève: Université de Genève.

"The paper will discuss some changes in Bolzano's definition of mathematics attested in several quotations from the *Beyträge*, *Wissenschaftslehre* and *Grössenlehre*: is mathematics a theory of forms or a theory of quantities? Several issues that are maintained throughout Bolzano's works will be distinguished from others that were accepted in the *Beyträge* and abandoned in the *Grössenlehre*. Changes will be interpreted as a consequence of the new logical theory of truth introduced in the *Wissenschaftslehre*, but also as a consequence of the overcome of Kant's terminology, and of the radicalization of Bolzano's anti-Kantianism. It will be argued that Bolzano's evolution can be understood as a coherent move, if one compares the criticism expressed in the *Beyträge* on the notion of quantity with a different and larger notion of quantity that Bolzano developed already in 1816. This discussion is based on the discovery that two unknown texts mentioned by Bolzano can be identified with works by von Spaun and Vieth respectively. Bolzano's evolution will be interpreted as a radicalization of the criticism of the Kantian definition of mathematics and as an effect of Bolzano's unaltered interest in the Leibnizian notion of *mathesis universalis*. As a conclusion, it will be argued that Bolzano never abandoned his original idea of considering mathematics as a *scientia universalis*, i.e. as the science of quantities in general, and it will be suggested that the question of ideal elements in mathematics, which has been interpreted as a main reason for the development of a new logical theory, can also be considered as a main reason for developing a different definition of quantity. "

44. Casari, Ettore. 1989. "Remarks on Bolzano's Modalities." In *Atti Del Convegno Internazionale Di Storia Della Logica: Le Teorie Della Modalità*, edited by Corsi, Giovanna, Mangione, Corrado and Mugnai, Massimo, 319-322. Bologna: CLUEB.

45. ———. 1992. "An Interpretation of Some Ontological and Semantical Notions in Bolzano's Logic." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 55-105. Firenze: Leo S. Olschki.
- "In the following, the attempt is done to clarify some significant features of Bolzano's logical system with particular attention to its development in the *Wissenschaftslehre* (WL). This system is viewed as a theory trying to identify certain quite general properties, relations and operations of *things* [*Dinge*], in the most general and unbiased sense of this word. For sake of simplicity and determinateness of the formulations, the current logical symbolism is *used*. The point of view of the theory is *elementary*, that is to say, we always work with *particular notions* about the things without allowing us any consideration of *arbitrary notions* about the things. 'x', 'y', 'z', ... are used as variables for things." (p. 55)
46. ———. 2006. "Some Remarks on Bolzano's Notion of a Quality." In *Logic and Philosophy in Italy. Some Trends and Perspectives. Essays in Honor of Corrado Mangione*, edited by Ballo, Edoardo and Franchella, Miriam, 185-201. Milano: Polimetrica.
47. Cavallin, Jens. 1997. *Content and Object. Husserl, Twardowski and Psychologism*. Dordrecht: Kluwer.
- Chapter Two: *Husserl and Bolzano*, pp. 69-82.
48. Cellucci, Carlo. 1992. "Bolzano and Multiple-Conclusion Logic." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 179-189. Firenze: Leo S. Olschki.
- "The aim of this paper is to assess Bolzano's logical work in the light of contemporary logical developments. This has been done before by others, most recently by van Benthem, (1) but everybody has his own approach and my approach -- whatever its value -- will be somewhat different from the current one. Make no mistake, I am not going to discuss once again to what extent Bolzano anticipated modern logic. On the contrary I will try to show how far he was from modern logic. In order to do so I will compare Bolzano with the tradition of multiple-conclusion logic." (p.179)
(1) J. van Benthem, *The Variety of Consequence, According to Bolzano*, *Studia Logica* 44, 1985, pp. 389-403.
49. Centrone, Stefania. 2010. "Functions in Frege, Bolzano and Husserl." *History and Philosophy of Logic* no. 31:315-336.
- "This explorative article is organized around a set of questions concerning the concept of a function. First, a summary of certain general facts about functions that are a common coin in contemporary logic is given. Then Frege's attempt at clarifying the nature of functions in his famous paper *Function and Concept* and in his *Grundgesetze* is discussed along with some questions which Frege's approach gave rise to in the literature. Finally, some characteristic uses of functional notions to be found in the work of Bernard Bolzano and in Edmund Husserl's early work are presented and elucidated."
50. Chattopadhyaya, Debi Prasad. 1979. "Bolzano and Frege: A Note on Ontology." In *Logic, Ontology and Action*, edited by Banerjee, K.K., 214-242. Atlantic Highlands: Humanities Press.
51. Chihara, Charles. 1999. "Frege's and Bolzano's Rationalist Conceptions of Arithmetic." *Revue d'Histoire des Sciences* no. 52 (3-4):343-361.
52. Chisholm, Roderick. 1989. "Bolzano on the Simplicity of the Soul." In *Traditionen Und Perspektiven Der Analytischen Philosophie*, edited by Gombocz, Wolfgang, Rute, Heiner and Sauer, Werner. Vienna: Hölder-Pichler-Tempsky.
53. ———. 1991. "Bernard Bolzano's Philosophy of Mind." *Philosophical Topics* no. 19:207-216.
- "According to Bolzano, the best way to find out what thinking as such requires is to reflect upon our own acts of thought. Such acts can be performed only by substances and the acts themselves do not logically require any multiplicity of parts. But thinkers may be corporeal in the sense of "having" bodies. The only thinker who does not have a body is God. Corporeal thinkers may be rational or non-rational. The so-called lower animals are non-rational. Such entities, unlike rational beings, cannot be called "minds". These conclusions are developed in detail in Bolzano's "Athanasia: Or Grounds for the Immortality of the

Soul". "

54. Chisholm, Roderick M. 1986. "On the Positive and Negative State of Things." In *Non-Existence and Predication*, edited by Haller, Rudolf, 97-106. Amsterdam: Rodopi.
55. Coffa, J. Alberto. 1982. "Kant, Bolzano and the Emergence of Logicism." *Journal of Philosophy* no. 74:679-689.

"Logicism is presented as an ingredient in a tradition designed to show that Kant's pure intuition plays no role in the foundation of "a priori" knowledge. Kant had argued for the necessity of the pure intuition from two assumptions: the claim that there is synthetic "a priori" knowledge and "the principle of synthetic judgments", (i.e., the thesis that synthetic judgments must be grounded on intuition). Bolzano was the first one to note that this principle is false and to propose that mathematics be constructed from concepts alone, thus excluding the representations that Kant associated with human sensibility."

56. ———. 1991. *The Semantic Tradition from Kant to Carnap*. Cambridge: Cambridge University Press.

See the Second Chapter: *Bolzano and the Birth of Semantics* pp. 22-40.

57. Cohen, Jonathan L. 1982. "Bolzano's Theory of Induction." In *Impact of Bolzano's Epoch on Development of Science - Conference Papers Prague 1981*, 443-457. Prague: Ustav ceskoslovenských a svetových dejin CSAV.
58. Corcoran, John. 1975. "Meanings of Implication." *Diálogos* no. 9:59-76.

Reprinted in: R. I. G. Hughes (ed.) - *A philosophical companion to First-order Logic* - pp. 85-100.

"In philosophical and mathematical discourse as well as in ordinary scholarly contexts the term 'implies' is used in several clear senses, many of which have already been noticed and explicated. The first five sections of this article codify and interrelate the most widely recognized meanings. Section 6 discusses a further significant and common use. Section 7 discusses and interrelates Tarski's notion of logical consequence, the "model-theoretic" notion of logical consequence, and Bolzano's two grounding relations. The eighth section employs the use-mention distinction to separate the three common grammatical categories of 'implies'. Section 8 also shows that criteria based on use-mention are not reliable indications of intended usage of 'implies'. The ninth and last section relates the above to the counterfactual and gives reasons for not expecting to find 'implies' used to express counterfactuals. A summary is provided.

Summary and Conclusion: In the first five sections we have distinguished twelve uses of the term 'implies'. At the outset we distinguished: implies1 (truth-functional), implies2 (logical consequence) and implies3 (logical deducibility). Next we distinguished three elliptical or enthymematic varieties of implication: C-implies1, C-implies2 and C-implies3. In none of these six senses did "A implies B" presuppose the truth of A. Then we discussed the cases wherein "A implies B" is used to mean "The-fact-that-A implies B," which does presuppose the truth of A. We paraphrased the latter as "A is true and A implies 13" where 'implies' indicates any of the previous six senses of the term. Thus, at that point, twelve senses of implies were distinguished, six which do not presuppose the truth of the implying sentence and six which do. Of the six which do, three are enthymematic.

In addition, the three original senses were carefully distinguished and interrelated, and possible causes of confusion were identified.

Then, building on some off-hand observations of Russell, we related the truth-functional use of 'implies' to two further notions which have been used as explications of traditional logical consequence. We also brought in Bolzano's relative implication and his two grounding relations.

We argued briefly that counterfactuals are not normally expressed using 'implies' and that the distinction between use and mention cannot be used as a test for distinguishing different meanings of 'implies'.

Use of 'implies' as a transitive verb taking a human subject has been ignored."

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