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## Annotated bibliography of Nino Cocchiarella 1978-1992

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### Essays 1978-1992

1. Cocchiarella, Nino. 1978. "On the Logic of Nominalized Predicates and Its Philosophical Interpretations." *Erkenntnis* no. 13:339-369.  
Errata, *Erkenntnis*, 14, 103-104, pp. 103-104.  
"Predicate nominalizations are transformations of predicates and predicate phrases into nouns or noun phrases. Thus, e.g., 'pious' is transformed into 'piety', 'wise' into 'wisdom', 'triangular' into 'triangularity', and 'human' into 'humanity'. We call these types of derivative nouns abstract singular terms. Some relational predicates are also transformed into abstract singular terms: e.g., 'identity' for 'is identical with' and 'indiscernibility' for 'is indiscernible from'.  
There are other forms which predicate nominalizations take as well. E.g., the noun phrase 'the concept Horse', especially as used by Frege, amounts to a nominalization of the predicate 'horse', and others of a related sort are 'the property red' and 'the relation of being taller than'. These nominalizations have stylistic variations, e.g., 'redness' or 'red' simpliciter (when used as a singular term rather than as a predicate) and 'the taller-than relation' or simply 'being taller than'.  
There are no doubt a number of distinctions relevant to linguistics that should be drawn between these different types of nominalizations. We, however, shall not pursue them here but shall concern ourselves instead with the more formal question of a logic of nominalized predicates in the context of some of its philosophical interpretations. We shall assume in this regard that the occurrences of nominalized predicates in ordinary discourse for which the logic is designed are all singular terms in the modern sense, i.e., that they purport to have singular reference in the same sense in which proper names and (unreduced) definite descriptions are said to have such reference." (p. 339)

2. ———. 1979. "The Theory of Homogeneous Simple Types as a Second Order Logic." *Notre Dame Journal of Formal Logic* no. 3:505-524.  
 "In its original form the theory of simple types, hereafter called ST, is a theory of predication and not, or at least not primarily, a theory of membership. With that original form in mind we construct in this paper a second order counterpart of ST which we call ST\*. We briefly compare ST\* with an alternative extension of second order logic, viz., the author's system T\*(\*) of [1], which was proposed as characterizing the original (and yet consistent!) logistic background of Russell's paradox of predication.  
 In [2], the author showed the completeness of T\*\*, plus an extensionality axiom (Ext\*), relative to a Fregean interpretation of subject-position occurrences of predicates, viz., that such occurrences of predicates denote individuals correlated with the properties (or "classes") designated by predicate-position occurrences of the same predicates. It is observed here that when the semantical Fregean frames characterized satisfy ST\*'s stratified comprehension principle instead of T\*\*'s general comprehension principle, then the same Fregean interpretation yields a completeness theorem for monadic ST\* + (Ext\*) as well. It has been found convenient, on the other hand, to consider (monadic) ST as a theory of membership rather than a theory of predication when axioms of extensionality are included in its characterization. So considered, Quine proposed his system NF as a first order counterpart of ST, though of course, as is well-known, NF far exceeds ST in deductive powers. We show here per contra that while (monadic) ST\* + (Ext\*) is motivated in its construction along lines followed by Quine in the construction of his first order counterpart NF, viz., the reduction of ST's metatheoretic feature of typical ambiguity to a stratified comprehension principle, our system, unlike NF, is equiconsistent with ST. This, along with the fact that the non-abstract individuals (or "urelements") of ST are retained unmodified in ST\*, indicates that ST\*, as a theory of predication, is to be preferred to NF, as a theory of membership, in the interpretation which each gives to ST's metatheoretic feature of typical ambiguity. We show in addition that if to (monadic) ST\* + (Ext\*) we add the assumption that whatever is a value of an individual variable is also (or, on the Fregean interpretation, is correlated with) a value of a (monadic) predicate variable, i.e., the assumption that every individual is a "class", then the resulting system is equiconsistent with NF. We refer to monadic ST\* + (Ext\*) as NFU\* and show that it contains Jensen's system NFU as well." (pp. 505-506)
- References  
 [1] Cocchiarella, N., "Whither Russell's paradox of predication?" in *Logic and Ontology*, M. K. Munitz, ed., New York University Press, New York (1973), pp. 133-158.  
 [2] Cocchiarella, N., "Fregean semantics for a realist ontology?" *Notre Dame Journal of Formal Logic*, vol. XV (1974), pp. 552-568.
3. ———. 1980. "Nominalism and Conceptualism as Predicative Second Order Theories of Predication." *Notre Dame Journal of Formal Logic* no. 21:481-500.  
 "There appears to be a growing consensus, even if not unanimity, that standard predicative second-order logic is the appropriate logical medium for the representation of a nominalist theory of predication. We agree that this is indeed the case and formulate in this paper a model-theoretic approach which justifies that claim. (1) Because it is model-theoretic, our approach differs from the truth-value semantics approach of Leblanc and Weaver. (2) Amongst other reasons, we prefer our model-theoretic approach so as to accommodate those nominalists for whom the assumption that there are potentially as many names as there are individuals is not acceptable.  
 The models involved in our semantics, moreover, are precisely the same models as are already involved in standard first-order logic. Assignments of values (drawn from the domain of a given model) to the individual variables are extended, however, to what, relative to a given first-order language, we call *nominalistic assignments* to the  $n$ -place predicate variables (for each positive integer  $n$ ) these

assign first-order formulas (wffs) of the language in question, relative to the free occurrences of  $n$  distinct individual variables occurring in those wffs, to the  $H$ -place predicate variables. The satisfaction by such an assignment of a second-order wff in a model is then defined by a double recursion on the logical structure of the wff and on the number of nested predicate quantifiers occurring therein.

It is natural of course that a first-order wff, relative to  $n$  individual variables occurring therein as argument indicators, should be understood as representing an  $n$ -place predicate expression of the language in question; and in fact in an applied first-order theory based upon that language such a first order wff would constitute the definiens of a possible definition for an  $n$ -place predicate constant not already belonging to that language or occurring in that theory. Potentially, of course, there are infinitely many predicate constants that might be introduced into a first-order theory in this way; and it is just over such a potential infinity, and no more, that our predicate quantifiers, nominalistically interpreted, are understood to range when we turn to the predicative second-order counterpart of a given first-order theory.

Finally, in order to better understand the implicit background of our nominalistic semantics, we include in a final section of this paper a brief comparison of nominalism, as represented by standard predicative second-order logic, with a closely related form of conceptualism, represented by a certain nonstandard predicative second-order logic formulated by the author in an earlier paper." (pp. 481-482)

(1) For the consensus view, see Parsons [9], For the dissenting view, at least in regard to the extension of predicative second-order logic to ramified type theory, see Church [2].

It is possible of course that Church intends his demurral to apply only after predicates are ramified and allowed to occur as subjects of higher-order predicates. If so, then we believe that his demurral may have some merit (see Note 10).

(2) For reasons indicated in Note 10, we suspect that ramification may presuppose a linguistic capacity for introducing predicates that exceeds the proper limits of a nominalist theory of predication. Such a capacity does not exceed the limits of a closely related form of conceptualism (briefly discussed in Section 6) which may be represented by the nonstandard predicative second-order logic formulated in [3].

References

[2] Church, A., "Comparison of Russell's Resolution of the Semantical Antinomies with that of Tarski," *The Journal of Symbolic Logic*, vol. 41 (1976), pp. 747-760.

[3] Cocchiarella, N., "A new formulation of predicative second order logic," *Logique et Analyse*, vol. 17, no. 65-66 (1974), pp. 61-88.

[9] Parsons, C., "A plea for substitutional quantification," *The Journal of Philosophy*, vol. 68 (1971), pp. 231-237.

4. ———. 1980. "The Development of the Theory of Logical Types and the Notion of a Logical Subject in Russell's Early Philosophy." *Synthese* no. 45:71-115.

Reprinted as Chapter 1 in *Logical Studies in Early Analytic Philosophy*, pp. 19-63.

"The development of the theory of logical types in Russell's early philosophy proceeds along a difficult and rather involuted path; and even the final product, the theory as adumbrated in [*Principia Mathematica* = PM], remains unclear in its syntax and problematic in its semantics. Indeed, one might well be left with the impression that Russell himself, in the end, remained unsure of which parts of the different views he had held along the way are finally to be adopted.

In what follows, we shall attempt to describe and explain the development of Russell's early views, at least to the extent to which they are available in published form today, from the perspective of the development in those views of the notion of a logical subject. It is the development of this notion in Russell's early philosophy, we believe, that holds the key to many of the problems confronting Russell in the development of his theory of logical types and that led to the various, and sometimes conflicting, proposals that he made along the way.

It should be noted, however, that in referring to the development of the theory of logical types in Russell's early philosophy we have in mind only the views

developed by Russell up to, but not subsequent to, the 1910—13 publication of the first edition of [PM]. The subsequent views developed by Russell from 1913—25, that is, between the first and second editions of [PM], and summarized to some extent in his introduction (and added appendices) to the second edition, constitute Russell's version of logical atomism. Except for some concluding remarks in the final section of this chapter, we delay our discussion of those views until chapter 5." (pp. 19-20 of the reprint)

5. ———. 1981. "Richard Montague and the Logical Analysis of Language." In *Contemporary Philosophy: A New Survey. Vol. 2: Philosophy of Language*, edited by Fløistad, Guttorm, 113-155. The Hague: Martinus Nijhoff.
- "Richard Montague was an exceptionally gifted logician who made important contributions in every field of inquiry upon which he wrote. His professional career was not only marked with brilliance and insight but it has become a classic example of the changing and developing philosophical views of logicians in general, especially during the 1960s and 70s, in regard to the form and content of natural language. We shall, in what follows, attempt to characterize the general pattern of that development, at least to the extent that it is exemplified in the articles Montague wrote during the period in question.
- The articles to which we shall especially direct our attention are: 'Pragmatics' [1]; 'Pragmatics and Intensional Logic' [2]; 'On the Nature of Certain Philosophical Entities' [3]; 'English as a Formal Language' [4]; 'Universal Grammar' [5]; and 'The Proper Treatment of Quantification in Ordinary English' [7].
- Needless to say, but many of the ideas and insights developed in these papers Montague shared with other philosophers and logicians, some of whom were his own students at the times in question. Montague himself was meticulous in crediting others where credit was due, but for convenience we shall avoid duplicating such references here." (p. 113)

(...)

#### "Concluding Remarks

There are many other important features of Montague's grammar for English and of his translation of English by means of that grammar into intensional logic that we cannot go into here. The highly intensional nature of his semantics, for example, provides not only a more direct analysis of the opacity of intensional verbs but also a more direct analysis of the opacity of infinitive phrases as well. And then there is his treatment of relative clauses and of attributive adjectives, which we have not touched upon at all.

In closing then, it will no doubt have crossed the reader's mind that there may be some irony in the fact that Montague began his philosophical career as an extensionalist who took set theory as the proper theoretical framework for philosophy and as a formal-language philosopher who viewed the formalization of ordinary language as either impossible or extremely laborious, and in any case as certainly not philosophically rewarding. For the fact is that Montague has made important and philosophically innovative contributions toward a fully formalized syntax and semantics for natural language and that the semantics in question is most perspicuously described in terms of an intensional logic that transcends set theory and that in effect constitutes a new theoretical framework for philosophy. If this is not a revolution, it is at least a form of progress in the logical analysis of language." (p. 155)

#### References

[I] Works by Richard Montague

(The first 9 articles are reprinted in *Formal Philosophy, Selected Papers of Richard Montague*, edited and with an introduction by R.H. Thomason, Yale University Press, New Haven 1974. All page references here are to this volume. The dates listed are not the dates of publication but of when Montague first presented each paper to a philosophical audience.)

[1] "Pragmatics," 1964.

[2] "Pragmatics and Intensional Logic," 1967.

- [3] "On the Nature of Certain Philosophical Entities,"1967.  
 [4] "English as a Formal Language,"1968.  
 [5] "Universal Grammar,"1969.  
 [6] "The Proper Treatment of Quantification in Ordinary English,"1970.  
 [7] "Syntactical Treatments of Modality,"1963.  
 [8] Montague and D. Kalish, "That,"1959.  
 [9] Montague and D. Kaplan, "A Paradox Regained,"1960.  
 [10] Montague and D. Kalish, *Logic: Techniques of Formal Reasoning*. New York: Harcourt, Brace and World, 1964.
6. ———. 1982. "Meinong Reconstructed versus Early Russell Reconstructed." *Journal of Philosophical Logic* no. 11:183-214.  
 Reprinted as Chapter 3 in *Logical Studies in Early Analytic Philosophy*, pp. 119-151.  
 "Contemporary philosophy is in a rut, according to Terence Parsons in his recent book *Nonexistent Objects*, ([NO]), and it is one that stems from the (post-1905) work of Bertrand Russell. The main characteristic of this "Russellian rut" ([NO], 1) is strict adherence to the thesis that being, or being something, amounts to being something that exists—or equivalently that 'there is' is to be equated with 'there exists' ([NO], 6). This view is now so well entrenched, according to Parsons, that it is a main stay of what he also calls the orthodox tradition.  
 Now the orthodox view is in a rut, according to Parsons, "because it's a view in which most of us are so entrenched that it's hard to see over the edges" ([NO], 1). Naturally, if we want "to look over the edge and see how things might be different" ([NO], 8), as any objective seeker of truth would, then "we need to encounter an actual theory about nonexistent objects" (ibid.). It is the construction and presentation of such a theory that is Parsons's concern in *Nonexistent Objects*.  
 (...)  
 "Now we do not object to Parsons's choice of Meinong's theory here, nor for that matter to his elegant reconstruction and presentation of that theory. We do think, however, that a more balanced recognition of Russell's overall view is called for and that perhaps the best way to make the Meinongian notion of a concrete object understandable to the orthodox tradition is to compare it with the general Russellian notion of a concrete individual, i.e., the Russellian notion of an individual that can exist but which might in fact not exist. Indeed, on the basis of the analysis and comparison we shall give here, it is our position that the Meinongian notion of a concrete object, at least as reconstructed by Parsons, is parasitic upon, though in a beneficent way, the Russellian notion of a concrete individual, existent or otherwise." (pp. 119-121)  
 References  
 [NO] Parsons, Terence, *Nonexistent Objects*, (New Haven and London: Yale University Press, 1980.)
7. ———. 1983. "Philosophical Perspectives on Quantification in Tense and Modal Logic." In *Handbook of Philosophical Logic. Vol. 2. Extensions of Classical Logic*, edited by Gabbay, Dov and Guentner, Franz, 309-353. Dordrecht: Reidel.  
 Reprinted in Dov M. Gabbay and F. Guentner, *Handbook of Philosophical Logic, Second Edition*, Vol. 7, Dordrecht: Springer 2002, pp. 235-275.  
 Contents: Introduction 235; 1. The Primary Semantics of Logical Necessity 236; 2. Logical Atomism and Quantified Modal Logic 237; 3. The Secondary Semantics of Metaphysical Necessity 240; 4. Proper Names as Rigid Designators 242; 5. Non-Contingent Identity and the Carnap-Barcan Formula 243; 6. Existence in the Primary and Secondary Semantics 245; 7. Metaphysical Necessity and Relational Model Structures 247; 8. Quantification with Respect to Individual Concepts 250; 9. Individual Concepts and the Elimination of *de re* Modalities 253; 10. Contingent Identity 258; 11. Quantifiers as Referential Concepts 258; 12. Singular Reference 259; 13. Conceptualism and Tense Logic 262; 14. The Problem of Reference to Past and Future Objects 266; 15. Time and Modality 268; Bibliography 274-275.

"The trouble with modal logic, according to its critics, is quantification into modal contexts - i.e. *de re* modality. For on the basis of such quantification, it is claimed, essentialism ensues, and perhaps a bloated universe of *possibilia* as well. The essentialism is avoidable, these critics will agree, but only by turning to a Platonic realm of individual concepts whose existence is no less dubious or problematic than mere *possibilia*. Moreover, basing one's semantics on individual concepts, it is claimed, would in effect render all identity statements containing only proper names either necessarily true or necessarily false - i.e. there would then be no contingent identity statements containing only proper names.

None of these claims is true quite as it stands, however; and in what follows we shall attempt to separate the chaff from the grain by examining the semantics of (first-order) quantified modal logic in the context of different philosophical theories. Beginning with the primary semantics of logical necessity and the philosophical context of logical atomism, for example, we will see that essentialism not only does not ensue but is actually rejected in that context by the validation of the modal thesis of anti-essentialism, and that in consequence all *de re* modalities are reducible to *de dicto* modalities.

(...)

Besides the Platonic view of intensionality, on the other hand, there is also a socio-biologically based conceptualist view according to which concepts are not independently existing Platonic forms but cognitive capacities or related structures of the human mind whose realization in thought is what informs a mental act with a predicable or referential nature. This view, it will be seen, provides an account in which there can be contingent identity statements, but not such as to depend on the coincidence of individual concepts in the platonic sense. Such a conceptualist view will also provide a philosophical foundation for quantified tense logic and paradigmatic analyses thereby of metaphysical modalities in terms of time and causation. The problem of the objective significance of the secondary semantics for the analyzed modalities, in other words, is completely resolved on the basis of the nature of time, local or cosmic. The related problem of a possible ontological commitment to *possibilia*, moreover, is in that case only the problem of how conceptualism can account for direct references to past or future objects." (pp. 235-236)

8. ———. 1985. "Two Lambda-Extensions of the Theory of Homogeneous Simple Types as a Second Order Logic." *Notre Dame Journal of Formal Logic* no. 26:377-407.

Contents:

0. Introduction 377; 1. HST\* revisited 379; 2. An improved axiom set for HST\* 380; 3. The grammar of HST\* with lambda-abstracts 383; 4. The system lambda-HST\* 384; 5. The system HST\*lambda 386; 6. An extensional Fregean semantics for nominalized predicates 390; 7. The relative consistency of HST\*-lambda + (Ext\*) to lambda-HST\* + (Ext\*) 393; 8. An intensional Fregean semantics for nominalized predicates 395; 9. Identity versus indiscernibility in HST\*-lambda 402; Notes 406; References 407.

Abstract: "Two second order logics with lambda-abstracts are formulated as counterparts to the theory of homogeneous simple types. Predicates can be nominalized and occur as abstract singular terms in these logics so that self-predication is meaningful in general and, in certain special cases, even provable. Extensional and intensional Fregean semantics in which nominalized predicates are assigned individuals as concept-correlates are formulated and the extensional and intensional versions of these logics are shown to be complete with respect to their corresponding semantics. The logics are also shown to be consistent relative to weak Zermelo set theory."

"In the theory of simple logical types as originally conceived, it is meaningless for one predicate expression to occur in one of the subject or argument positions of another unless the latter is assigned a higher logical type than the former within the grammar of the object language; and therefore it is meaningless in particular for any

predicate expression to apply to itself, i.e., to occur in one of its own subject positions. Russell's paradox of predication is thereby avoided, of course, but the price is high, for the resulting theory is not an accurate representation of the role of predicates in natural language where predicate expressions can apply not only to the nominalizations of other predicates but to their own nominalizations as well -- and without regard at all for the notion of a logical type. In the theory of logical types as a second-order logic, on the other hand, predicate expressions are typed within the grammar of the object language only in the way they are typed in standard second-order logic, i.e., only with respect to their degree or number of subject positions, and they are allowed otherwise to meaningfully occur in the subject or argument positions of other predicates, and of themselves as well, without regard to the notion of a logical type. Russell's paradox of predication can be avoided, it turns out, not by resorting to the notion of a logical type as a part of the grammar of the object language but rather only as a part of the metalinguistic description of the conditions under which properties and relations are to be posited by means of the grammar of the object language. The difference is crucial, needless to say, since it allows for a more accurate representation of the role of predicates and predication in natural language. The resulting theory is not, to be sure, a second-order logic in the "standard" sense used today (though it does contain the latter), but it is a second-order logic in the traditional or pre-type-theoretical sense in which quantifier expressions are allowed to reach into both subject and predicate positions without obliterating the logical and conceptually important distinctions between the two." (pp. 377-378)

9. ———. 1985. "Frege's Double Correlation Thesis and Quine's Set Theories NF and ML." *Journal of Philosophical Logic* no. 14:1-39.  
Reprinted as Chapter 4 in *Logical Studies in Early Analytic Philosophy*, pp. 152-192.

"There are two fundamentally different notions of a class, which, following tradition, we might call the mathematical and the logical notions, respectively. The logical notion is essentially the notion of a class as the extension of a concept, and, following Frege, we shall assume that a class in this sense "simply has its being in the concept, not in the objects which belong to it" (Frege, [PW], 183)—regardless of whether or not concepts themselves differ, as Frege assumed, "only so far as their extensions are different" (ibid., 118). The mathematical notion of a class, on the other hand, is essentially the notion of a class as composed of its members, i.e., of a class that has its being in the objects that belong to it. This notion of a class, we claim, is none other than the iterative concept of set—or at least that is what it comes to upon analysis. Note that although what accounts for the being of a class under the one notion is not the same as what accounts for the being of a class under the other, nevertheless the axiom of extensionality applies equally to both notions. This means that the axiom of extensionality does not of itself account for the being of a class. (1)

Of course the logical notion of a class, especially as developed in Frege's form of logicism, is usually thought to be bankrupt as a result of Russell's paradox. This assessment, however, is erroneous. In particular, in "Frege, Russell, and Logicism: A Logical Reconstruction," ([FRL]), I have explained how Frege's view of classes in the logical sense can be reconstructed without paradox by modifying in either of two ways what I there referred to as Frege's double correlation thesis. The two systems that result from these modifications, it turns out, have certain structural similarities with Quine's two set theories NF and ML, especially when the latter are themselves modified so as to include urelements other than the empty set. This is significant because both NF and ML are commonly said to "lack a motivation" (cf. Boolos's "The Iterative Concept of Set" ([ICS]), 219). But that is because as theories of sets in the sense of classes that are composed of their members, which is really the only sense to which Quine is willing to commit himself, both NF and ML are incompatible with the iterative concept of set. As theories of classes in the logical sense, however, and in particular of the classes that Frege took to be the

correlates of concepts, both NF and ML can be given a very natural motivation, especially when modified to include urelements. In what follows we shall defend this motivation by examining the structural similarities in question." (pp. 152-153 of the reprint)

(1) In an intensional language, the mathematical notion of a class might well assume a stronger axiom of extensionality, viz. one in which classes that are composed of their members are necessarily identical when they have the same members. Such an axiom would not in general hold for classes in the logical sense, since co-extensive concepts are not in general necessarily co-extensive. (It would of course hold for those classes in the logical sense that are the extensions of "rigid" concepts, i.e., concepts that have the same extension in every possible world.)

References

[ICS] G. Boolos, "The Iterative Conception of Set," *Journal of Philosophy* 68 (1971):215—31.

[PW] G. Frege, *Posthumous Writings*, eds. H. Hermes, F. Kambartel and F. Kaulbach; translated by P. Long and R. White (Oxford: Blackwell, 1979).

10. ———. 1986. "Frege, Russell and Logicism: A Logical Reconstruction." In *Frege Synthesized: Essays on the Philosophical and Foundational Work of Gottlob Frege*, edited by Haaparanta, Leila and Hintikka, Jaakko, 197-252. Dordrecht: Reidel. Reprinted as Chapter 2 in *Logical Studies in Early Analytic Philosophy*, pp. 64-118. "Logicism by the end of the nineteenth century was a philosophical doctrine whose time had come, and it is Gottlob Frege to whom we owe its arrival. "Often," Frege once wrote, "it is only after immense intellectual effort, which may have continued over centuries, that humanity at last succeeds in achieving knowledge of a concept in its pure form, in stripping off the irrelevant accretions which veil it from the eyes of the mind" (Frege, *The Foundations of Arithmetic*, [Fd], xix). Prior to Frege logicism was just such a concept whose pure form was obscured by irrelevant accretions; and in his life's work it was Frege who first presented this concept to humanity in its pure form and developed it as a doctrine of the first rank. That form, unfortunately, has become obscured once again. For today, as we approach the end of the twentieth century, logicism, as a philosophical doctrine, is said to be dead, and even worse, to be impossible. Frege's logicism, or the specific presentation he gave of it in *Die Grundgesetze der Arithmetik*, ([Gg]), fell to Russell's paradox, and, we are told, it cannot be resurrected. Russell's own subsequent form of logicism presented in [PM], moreover, in effect gives up the doctrine; for in overcoming his paradox, Russell was unable to reduce classical mathematics to logic without making at least two assumptions that are not logically true; namely, his assumption of the axiom of reducibility and his assumption of an axiom of infinity regarding the existence of infinitely many concrete or nonabstract individuals. Contrary to popular opinion, however, logicism is not dead beyond redemption; that is, if logicism is dead, then it can be easily resurrected. This is not to say that as philosophical doctrines go logicism is true, but only that it can be logically reconstructed and defended or advocated in essentially the same philosophical context in which it was originally formulated. This is true especially of Frege's form of logicism, as we shall see, and in fact, by turning to his correspondence with Russell and his discussion of Russell's paradox, we are able to formulate not only one but two alternative reconstructions of his form of logicism, both of which are consistent (relative to weak Zermelo set theory). In regard to Russell's form of logicism, on the other hand, our resurrection will not apply directly to the form he adopted in [PM] but rather to the form he was implicitly advocating in his correspondence with Frege shortly after the completion of [POM]. In this regard, though we shall have occasion to refer to certain features of his later form of logicism, especially in our concluding section where a counterpart to the axiom of reducibility comes into the picture, it is Russell's early form of logicism that we shall reconstruct and be concerned with here.

Though Frege's and Russell's early form of logicism are not the same, incidentally, they are closely related; and one of our goals will be to reconstruct or resurrect these forms with their similarity in mind. In particular, it is our contention that both are to be reconstructed as second order predicate logics in which nominalized predicates are allowed to occur as abstract singular terms. Their important differences, as we shall see, will then consist in the sort of object each takes nominalized predicates to denote and in whether the theory of predication upon which the laws of logic are to be based is to be extensional or intensional." (pp. 64-65 of the reprint)

#### References

Frege, Gottlob, [Fd] *The Foundations of Arithmetic*, trans, by J. L. Austin, Harper & Bros., N.Y. 1960.

Frege, Gottlob, [Gg] *Die Grundgesetze der Arithmetik*, vols. 1 and 2, Hildesheim, 1962.

Russell, Bertrand, [PM] *Principia Mathematica*, coauthor, A. N. Whitehead, Cambridge University Press, 1913.

Russell, Bertrand, [POM] *The Principles of Mathematics*, 2nd ed., W. W. Norton & Co., N.Y., 1937.

11. ———. 1986. "Conceptualism, Ramified Logic, and Nominalized Predicates." *Topoi. An International Review of Philosophy* no. 5:75-87.

"The problem of universals as the problem of what predicates stand for in meaningful assertions is discussed in contemporary philosophy mainly in terms of the opposing theories of nominalism and logical realism. Conceptualism, when it is mentioned, is usually identified with intuitionism, which is not a theory of predication but a theory of the activity of constructing proofs in mathematics. Both intuitionism and conceptualism are concerned with the notion of a mental construction, to be sure, and both maintain that there can only be a potentially infinite number of such constructions. But whereas the focus of concern in intuitionism is with the construction of proofs, in conceptualism our concern is with the construction of concepts. This difference sets the two frameworks apart and in pursuit of different goals, and in fact it is not at all clear how the notion of a mental construction in the one framework is related to that in the other. This is especially true insofar as mathematical objects, according to intuitionism, are nothing but mental constructions, whereas in conceptualism concepts are anything but objects. In any case, whatever the relation between the two, our concern in this paper is with conceptualism as a philosophical theory of predication and not with intuitionism as a philosophy of mathematics.

Now conceptualism differs from nominalism insofar as it posits universals, namely, concepts, as the semantic grounds for the correct or incorrect application of predicate expressions. Conceptualism differs from logical realism, on the other hand, insofar as the universals it posits are not assumed to exist independently of the human capacity for thought and representation. Concepts, in other words, are neither predicate expressions nor independently real properties and relations. But then, at least for the kind of conceptualism we have in mind here, neither are they mental images or ideas in the sense of particular mental occurrences. That is, concepts are not objects (saturated individuals) but are rather cognitive capacities, or cognitive structures otherwise based upon such capacities, to identify and classify or characterize and relate objects in various ways. Concepts, in other words, are intersubjectively realizable cognitive abilities which may be exercised by different persons at the same time as well as by the same person at different times. And it is for this reason that we speak of concepts as objective universals, even though they are not independently real properties and relations.

As cognitive structures, however, concepts in the sense intended here are not Fregean concepts (which for Frege are independently real unsaturated functions from objects to truth values). But they may be modeled by the latter (assuming that there are Fregean concepts to begin with) -especially since as cognitive capacities which need not be exercised at any given time (or even ever for that matter), concepts in the sense intended here also have an unsaturated nature corresponding

to, albeit different from, the unsaturated nature of Fregean concepts. Thus, in particular, the saturation (or exercise) of a concept in the sense intended here results not in a truth value but a mental act, and, if overtly expressed, a speech act as well. The un-saturatedness of a concept consists in this regard in its non-occurrent or purely dispositional status as a cognitive capacity, and it is the exercise (or saturation) of this capacity as a cognitive structure which informs particular mental acts with a predicable nature (or with a referential nature in the case of concepts corresponding to quantifier expressions)." (pp. 75-76)

12. ———. 1987. "Rigid Designation." In *Encyclopedic Dictionary of Semiotics. Vol. 2*, edited by Sebeok, Thomas A., 834. Berlin: Mouton de Gruyter.
13. ———. 1987. "Russell, Bertrand." In *Encyclopedic Dictionary of Semiotics. Vol. 2*, edited by Sebeok, Thomas A., 840-841. Berlin: Mouton de Gruyter.

14. ———. 1988. "Predication Versus Membership in the Distinction between Logic as Language and Logic as Calculus." *Synthese* no. 75:37-72.  
Contents: 0. Introduction; 1. The problem with a set-theoretic semantics of natural language; 2. Intensional logic as a new theoretical framework for philosophy; 3. The incompleteness of intensional logic when based on membership; 4. Predication versus membership in type theory; 5. Second order predicate logic with nominalized predicates; 6. A set theoretic semantics with predication as fundamental; 7. Concluding remarks.

"There are two major doctrines regarding the nature of logic today. The first is the view of logic as the laws of valid inference, or logic as calculus. This view began with Aristotle's theory of the syllogism, or syllogistic logic, and in time evolved first into Boole's algebra of logic and then into quantificational logic. On this view, logic is an abstract calculus capable of various interpretations over domains of varying cardinality. Because these interpretations are given in terms of a set-theoretic semantics where one can vary the universe at will and consider the effect this, has on the validity of formulas, this view is sometimes described as the set-theoretic approach to logic (see van Heijenoort ["Logic as Language and Logic as Calculus", *Synthese* 17,] 1967, p. 327).

The second view of logic does not eschew set-theoretic semantics, it should be noted, and it may in fact utilize such a semantics as a guide in the determination of validity. But to use such a semantics as a guide, on this view, is not the same as to take that semantics as an essential characterization of validity. Indeed, unlike the view of logic as calculus, this view of logic rejects the claim that a set-theoretic definition of validity has anything other than an extrinsic significance that may be exploited for certain purposes (such as proving a completeness theorem). Instead, on this view, logic has content in its own right and validity is determined by what are called the laws of logic, which may be stated either as principles or as rules. Because one of the goals of this view is a specification of the basic laws of logic from which the others may be derived, this view is sometimes called the axiomatic approach to logic." (p. 37)

(...)

"Concluding Remarks. The account we have given here of the view of logic as language should not be taken as a rejection of the set-theoretical approach or as defense of the metaphysics of possibilist logical realism. Rather, our view is that there are really two types of conceptual framework corresponding to our two doctrines of the nature of logic. The first type of framework is based on membership in the sense of the iterative concept of set; although extensionality is its most natural context (since sets have their being in their members), it may nevertheless be extended to include intensional contexts by way of a theory of senses (as in Montague's sense-denotation intensional logic). The second type of framework is based on predication, and in particular developments it is associated with one or another theory of universals. Extensionality is not the most natural context in this theory, but where it does hold and extensions are posited, the extensions are classes in the logical and not in the mathematical sense.

Russell's paradox, as we have explained, has no real bearing on set-formation in a theory of membership based on the iterative concept of set, but it does bear directly on concept-formation or the positing of universals in a theory based on predication. As a result, our second type of framework has usually been thought to be incoherent or philosophically bankrupt, leaving us with the set-theoretical approach as, the only viable alternative. This is why so much of analytic philosophy in the 20th Century has been dominated by the set-theoretical approach. Set theory, after all, does seem to serve the purposes of a *mathesis universalis*.

What is adequate as a *mathesis universalis*, however, need not also therefore be adequate as a *lingua philosophica* or *characteristica universalis*. In particular, the set-theoretic approach does not seem to provide a philosophically satisfying semantics for natural language; this is because it is predication and not membership that is fundamental to natural language. An adequate semantics for natural language, in other words, seems to demand a conceptual framework based on predication and not on membership.

(...)

We do not maintain, accordingly, that we should give up the set-theoretic approach, especially when dealing with the philosophy and foundations of mathematics, or that only a theory of predication associated with possibilist logical realism will provide an adequate semantics for natural language. In both cases we may find a principle of tolerance, if not outright pluralism, the more appropriate attitude to take." (pp. 69-70)

15. ———. 1989. "Philosophical Perspectives on Formal Theories of Predication." In *Handbook of Philosophical Logic. Vol. 4. Topics in the Philosophy of Language*, edited by Gabbay, Dov and Guentner, Franz, 253-326. Dordrecht: Reidel. Contents: 1. Predication and the problem of universal 254; 2. Nominalism 256; 3. A nominalistic semantics for predicative second order logic 261; 4. Nominalism and modal logic 266; 5 . Conceptualism vs . nominalism 270; 6. Constructive conceptualism 273; 7. Ramification of constructive conceptualism 280; 8. Holistic conceptualism 286; 9. Logical realism vs holistic conceptualism 289; 10. Possibilism and actualism in modal logical realism 292; 11. Logical realism and essentialism 301; 12. Possibilism and actualism within conceptualism 306; 13. Natural realism and conceptualism 313; 14. Aristotelian essentialism and the logic of natural kinds 318; References 325-326.
- "Predication has been a central, if not the central, issue in philosophy since at least the lime of Plato and Aristotle. Different theories of predication have in fact been the basis of a number of philosophical controversies in both metaphysics and epistemology, not the least of which is the problem of universals. In what follows we shall be concerned with what traditionally have been the three most important types of theories of universals. namely, nominalism, conceptualism, and realism, and with the theories of predication which these theories might be said to determine or characterize.
- Though each of these three types of theories of universals may be said to have many variants, we shall ignore their differences here to the extent that they do not characterize different theories of predication. This will apply especially to nominalism where but one formal theory of predication is involved. In both conceptualism and realism, however, the different variants of each type do not all agree and form two distinct subtypes each with its own theory of predication. For this reason we shall distinguish between a constructive and a holistic form of conceptualism on the one hand, and a logical and a natural realism on the other. Constructive conceptualism, as we shall see, has affinities with nominalism with which it is sometimes confused, and holistic conceptualism has affinities with logical realism with which it is also sometimes confused. Both forms of conceptualism may assume some form of natural realism as their causal ground; and natural realism in turn must presuppose some form of conceptualism as its background theory of predication. Both forms of realism may be further divided into their essentialist and non-essentialist variants (and in logical realism even a form of

anti-essentialism), and though an essentialist logical realism is sometimes confused with Aristotelian essentialism, the latter is really a form of natural realism with natural kinds as the only essential properties objects can have." (pp. 253-254)

16. ———. 1989. "Russell's Theory of Logical Types and the Atomistic Hierarchy of Sentences." In *Rereading Russell: Essays on Bertrand Russell's Metaphysics and Epistemology*, edited by Savage, C. Wade and Anderson, C. Anthony, 41-62. Minneapolis: University of Minnesota Press.  
Reprinted as Chapter 5 in *Logical Studies in Early Analytic Philosophy*, pp. 193-221.

"Russell's philosophical views underwent a number of changes throughout his life, and it is not always well-appreciated that views he held at one time came later to be rejected; nor, similarly, that views he rejected at one time came later to be accepted. It is not well-known, for example, that the theory of logical types Russell described in his later or post-[PM] philosophy is not the same as the theory originally described in [PM] in 1910-13; nor that some of the more important applications that Russell made of the theory at the earlier time cannot be validated or even significantly made in the framework of his later theory. What is somewhat surprising, however, is that Russell himself seems not to have realized that he was describing a new theory of logical types in his later philosophy, and that as a result of the change some of his earlier logical constructions, including especially his construction of the different kinds of numbers, were no longer available to him. In the original framework, for example, propositional functions are independently real properties and relations that can themselves have properties and relations of a higher order/type, and all talk of classes, and thereby ultimately of numbers, can be reduced to extensional talk of properties and relations as "single entities," or what Russell in [POM] had called "logical subjects." The Platonic reality of classes and numbers was replaced in this way by a more fundamental Platonic reality of propositional functions as properties and relations. In Russell's later philosophy, however, "a propositional function is nothing but an expression. It does not, by itself, represent anything. But it can form part of a sentence which does say something, true or false" (Russell, *My Philosophical Development*, ([MPD]), 69). Surprisingly, Russell even insists that this was what he meant by a propositional function in [PM]. "Whitehead and I thought of a propositional function as an expression containing an undetermined variable and becoming an ordinary sentence as soon as a value is assigned to the variable: 'x is human', for example, becomes an ordinary sentence as soon as we substitute a proper name for V. In this view . . . the propositional function is a method of making a bundle of such sentences" ([MPD], 124). Russell does realize that some sort of change has come about, however, for he admits, "I no longer think that the laws of logic are laws of things; on the contrary, I now regard them as purely linguistic" (ibid., 102).

(...)

Now it is not whether [PM] can sustain a nominalistic interpretation that is our concern in this essay, as we have said, but rather how it is that Russell came to be committed in his later philosophy to the atomistic hierarchy and the nominalistic interpretation of propositional functions as expressions generated in a ramified second order hierarchy of languages based on the atomistic hierarchy. We shall pursue this question by beginning with a discussion of the difference between Russell's 1908 theory of types and that presented in [PM] in 1910. This will be followed by a brief summary of the ontology that Russell took to be implicit in [PM], and that he described in various publications between 1910 and 1913. The central notion in this initial discussion is what Russell in his early philosophy called the notion of a logical subject, or equivalently that of a "term" or "single entity". (In [PM], this notion was redescribed as the systematically ambiguous notion of an "object.") As explained in chapter 1 this notion provides the key to the various problems that led Russell in his early philosophy to the development of his different theories of types, including that presented in [PM]. This remains true, moreover, even when we turn to Russell's later philosophy, i.e., to his post-[PM] views, only

then it is described as the notion of what can and cannot be named in a logically perfect language. The ontology of these later views is what Russell called logical atomism, and it is this ontology that determines what Russell described as the atomistic hierarchy of sentences. In other words, it is the notion of what can and cannot be named in the atomistic hierarchy that explains how Russell, however unwittingly, came to replace his earlier theory of logical types by the theory underlying the atomistic hierarchy of sentences as the basis of a logically perfect language." (pp. 193-195 of the reprint)

References

[POM] Russell, Bertrand, *The Principles of Mathematics*, 2d ed. (NY., Norton & Co., 1938).

[PM] Russell, Bertrand and Alfred Whitehead, *Principia Mathematica*, vol. 1 (1910), vol. 2 (1912), and vol. 3 (1913) (London: Cambridge Univ. Press,).

17. ———. 1989. "Conceptualism, Realism and Intensional Logic." *Topoi. An International Review of Philosophy* no. 7:15-34.  
 Contents: 0. Introduction 15; 1. A conceptual analysis of predication 16; 2. Concept-correlates and Frege's double correlations thesis 17; 3. Russell's paradox in conceptual realism 18; 4. What are the natural numbers and where do they come from? 22; 5. Referential concepts and quantifier phrases 24; 5. Singular reference 24; 7. The intensions of referential concepts as components of applied predicable concepts 26; 8. Intensional versus extensional predicable concepts 28; 9. The intentional identity of intensional objects 29; Notes 31; Reference 33-35.

"0. Introduction

Linguists and philosophers are sometimes at odds in the semantical analysis of language. This is because linguists tend to assume that language must be semantically analyzed in terms of mental constructs, whereas philosophers tend to assume that only a platonic realm of intensional entities will suffice. The problem for the linguist in this conflict is how to explain the apparent realist posits we seem to be committed to in our use of language, and in particular in our use of infinitives, gerunds and other forms of nominalized predicates. The problem for the philosopher is the old and familiar one of how we can have knowledge of independently real abstract entities if all knowledge must ultimately be grounded in psychological states and processes. In the case of numbers, for example, this is the problem of how mathematical knowledge is possible. In the case of the intensional entities assumed in the semantical analysis of language, it is the problem of how knowledge of even our own native language is possible, and in particular of how we can think and talk to one another in all the ways that language makes possible.

I believe that the most natural framework in which this conflict is to be resolved and which is to serve as the semantical basis of natural language is an intensional logic that is based upon a conceptual analysis of predication in which what a predicate stands for in its role as a predicate is distinguished from what its nominalization denotes in its role as a singular term. Predicates in such a framework stand for concepts as cognitive capacities to characterize and relate objects in various ways, i.e. for dispositional cognitive structures that do not themselves have an individual nature, and which therefore cannot be the objects denoted by predicate nominalizations as abstract singular terms. The objects purportedly denoted by nominalized predicates, on the other hand, are intensional entities, e.g. properties and relations (and propositions in the case of zero-place predicates), which have their own abstract form of individuality, which, though real, is posited only through the concepts that predicates stand for in their role as predicates. That is, intensional objects are represented in this logic as concept-correlates, where the correlation is based on a logical projection of the content of the concepts whose correlates they are.

(...)

Before proceeding, however, there is an important distinction regarding the notion of a logical form that needs to be made when joining conceptualism and realism in this way. This is that logical forms can be perspicuous in either of two senses, one

stronger than the other. The first is the usual sense that applies to all theories of logical form, conceptualist or otherwise; namely, that logical forms are perspicuous in the way they specify the truth conditions of assertions in terms of the recursive operations of logical syntax. In this sense, fully applied logical forms are said to be semantic structures in their own right. In the second and stronger sense, logical forms may be perspicuous not only in the way they specify the truth conditions of an assertion, but in the way they specify the cognitive structure of that assertion as well. To be perspicuous in this sense, a logical form must provide an appropriate representation of both the referential and the predicable concepts that underlie an assertion.

Our basic hypothesis in this regard will be that every basic assertion is the result of applying just one referential concept and one predicable concept, and that such an applied predicable concept is always fully intensionalized (in a sense to be explained). This will place certain constraints on the conditions for when a complex predicate expression is perspicuous in the stronger sense — such as that a referential expression can occur in such a predicate expression only in its nominalized form. (A similar constraint will also apply to a defining or restricting relative clause of a referential expression.) In the cases where a relational predicable concept is applied, the assumption that there is still but one referential concept involved leads to the notion of a conjunctive referential concept, a notion that requires the introduction in intensional logic of special quantifiers that bind more than one individual variable. Except for briefly noting the need for their development, we shall not deal with conjunctive quantifiers in this essay." (pp. 15-16)

18. ———. 1991. "Conceptualism." In *Handbook of Metaphysics and Ontology*, edited by Smith, Barry and Burkhardt, Hans, 168-174. Munich: Philosophia Verlag. Conceptualism is one of the three types of theories regarding the nature of universals described by Porphyry in his introduction to Aristotle's *Categories*. The other two are nominalism and realism. Because a universal, according to Aristotle, is that which can be predicated of things (De Int. 17a39), the difference between these three types of theories lies in what it is that each takes to be predicable of things. In this regard we should distinguish predication in language from predication in thought, and both from predication in reality, where there is no presumption that one kind of predication precludes the others.

All three types of theories agree that there is predication in language, in particular that predicates can be predicated of things in the sense of being true or false of them. Nominalism goes further in maintaining that only predicates can be predicated of things, that is, that there are no universals other than the predicate expressions of some language or other. Conceptualism opposes nominalism in this regard and maintains that predicates can be true or false of things only because they stand for concepts, where concepts are the universals that are the basis of predication in thought. Realism also opposes nominalism in maintaining that there are real universals, viz. properties and relations, that are the basis of predication in reality." (p. 168)

(...)

"Conceptualism is by no means a monolithic theory, but has many forms, some more restrictive than others, depending on the mechanisms assumed as the basis for concept-formation. None of these forms, in themselves, precludes being combined with a realist theory, whether Aristotelian (as in conceptual natural realism) or Platonist (as in conceptual intensional realism), or both. Some conceptualists, such as Sellars, have made it a point to disassociate conceptualism from any form of realism regarding abstract entities, but that disassociation has nothing to do with conceptualism as a theory about the nature of predication in thought.

Conceptualism's shift in emphasis from metaphysics to psychology, in other words, while important in determining what kind of theory is needed to explain predication in thought, should not be taken as justifying a restrictive form of conceptualism that precludes both a natural and an intensional realism." (p. 174)

19. ———. 1991. "Logic V: Higher Order Logics." In *Handbook of Metaphysics and Ontology*, edited by Smith, Barry and Burkhardt, Hans, 466-470. Munich: Philosophia Verlag.  
 "Higher-order logic goes beyond first-order logic in allowing quantifiers to reach into the predicate as well as the subject positions of the logical forms it generates. A second feature, usually excluded in standard formulations of second-order logic, allows nominalized forms of predicate expressions (simple or complex) to occur in its logical forms as abstract singular terms. (E.g., 'Socrates is wise', in symbols  $W(s)$ , contains 'is wise' as a predicate, whereas 'Wisdom is a virtue', in symbols  $V(W)$ , contains 'wisdom' as a nominalized form of that predicate. 'Being a property is a property', in symbols  $P(P)$ , or with  $\lambda$ -abstracts,  $P\lambda xP(x)$ ), where  $\lambda\chi P(\chi)$  is read 'to be an  $x$  such that  $x$  is a property', contains both the predicate 'is a property' and a nominalized form of that predicate, viz. 'being a property'. Frege's well-known example, 'The concept Horse is not a concept', contains 'the concept Horse' as a nominalized form of the predicate phrase 'is a horse'.)" (p. 466)
20. ———. 1991. "Ontology, Formal." In *Handbook of Metaphysics and Ontology*, edited by Smith, Barry and Burkhardt, Hans, 640-647. Munich: Philosophia Verlag.  
 "Formal ontology is the result of combining the intuitive, informal method of classical ontology with the formal, mathematical method of modern symbolic logic, and ultimately identifying them as different aspects of one and the same science. That is, where the method of ontology is the intuitive study of the fundamental properties, modes, and aspects of being, or of entities in general, and the method of modern symbolic logic is the rigorous construction of formal, axiomatic systems, formal ontology, the result of combining these two methods, is the systematic, formal, axiomatic development of the logic of all forms and modes of being. As such, formal ontology is a science prior to all others in which particular forms, modes, or kinds of being are studied." (p. 641)
21. ———. 1991. "Russell, Bertrand." In *Handbook of Metaphysics and Ontology*, edited by Smith, Barry and Burkhardt, Hans, 796-798. Munich: Philosophia Verlag.  
 "Russell held a number of different metaphysical positions throughout his career, with the idea of logic as a logically perfect language being a common theme that ran through each.  
 (...)  
 "A fundamental notion of Russell's logical realism, sometimes also called ontological logicism, was that of a propositional function, the extension of which Russell took to be a class as many. Initially, as part of his response to the problem of the One and the Many, Russell had assumed that each propositional function was a single and separate entity over and above the many propositions that were its values, and, similarly, that to each class as many there corresponded a class as one. Upon discovering his paradox, Russell maintained that we must distinguish a class as many from a class as one, and that a class as one might not exist corresponding to a class as many. He also concluded that a propositional function cannot survive analysis after all, but 'lives' only in the propositions that are its values, i.e. that propositional functions are nonentities."  
 (...)  
 "As a result of arguments given by Ludwig Wittgenstein in 1913, Russell, from 1914 on, gave up the Platonistic view that properties and relations could be logical subjects. Predicates were still taken as standing for properties and relations, but only in their role as predicates; i.e., nominalized predicates were no longer allowed as abstract singular terms in Russell's new version of his logically perfect language. Only particulars could be named in Russell's new metaphysical theory, which he called logical atomism, but which, unlike his earlier 1910-13 theory, is a form of natural realism, and not of logical realism, since now the only real properties and relations of his ontology are the simple material properties and relations that are the components of the atomic facts that make up the world. Complex properties and relations in this framework are simply propositional functions, which, along with propositions, are now merely linguistic expressions. (Russell remained unaware that

as a result of the change in his metaphysical views from logical to natural realism his original theory of types was restricted to the much weaker sub-theory of ramified second-order logic, and that he could no longer carry through his logicist programme. This reinforced the confusion of nominalists into thinking that Russell's earlier theory of types could be given a nominalistic interpretation, since such an interpretation is possible for ramified second-order logic.)" (pp. 797-798)

22. ———. 1991. "Quantification, Time and Necessity." In *Philosophical Applications of Free Logic*, edited by Lambert, Karel, 242-256. New York: Oxford University Press.
- Contents: 0. Introduction; 1. A Logic a Actual and Possible Objects; 2. A Completeness Theorem for Tense Logic; 3. Modality Within Tense Logic; 4. Some Observations on Quantifiers in Modal and Tense Logic; 5. Concluding Remarks.
- Abstract: "A logic of actual and possible objects is formulated in which "existence" and "being", as second-level concepts represented by first-order (objectual) quantifiers, are distinguished. A free logic of actual objects is then distinguished as a subsystem of the logic of actual and possible object. Several complete first-order tense logics are then formulated in which temporal versions of possibilism and actualism are characterized in terms of the free logic of actual objects and the wide logic of actual and possible objects. It is then shown how a number of different modal logics can be interpreted within quantified tense logic, with the latter providing a paradigmatic framework in which to distinguish the interplay between quantifiers, tenses and modal operators and within which we can formulate different temporal versions of actualism and possibilism."
- "The fundamental assumption of a logic of actual and possible objects is that the concept of existence is not the same as the concept of being. Thus, even though necessarily whatever exists has being, it is not necessary in such a logic that whatever has being exists; that is, it can be the case that there be something that does not exist. No occult doctrine is needed to explain the distinction between existence and being, for an obvious explanation is already at hand in a framework of tense logic in which being encompasses past, present, and future objects (or even just past and present objects) while existence encompasses only those objects that presently exist. We can interpret modality in such a framework, in other words, whereby it can be true to say that some things do not exist. Indeed, as indicated in Section 3, infinitely many different modal logics can be interpreted in the framework of tense logic. In this regard, we maintain, tense logic provides a paradigmatic framework in which possibilism (i.e., the view that existence is not the same as being, and that therefore there can be some things that do not exist) can be given a logically perspicuous representation.
- Tense logic also provides a paradigmatic framework for actualism as the view that is opposed to possibilism; that is, the view that denies that the concept of existence is different from the concept of being. Indeed, as we understand it here, actualism does not deny that there can be names that have had denotations in the past but that are now denotationless, and hence that the statement that some things do not exist can be true in a semantic metalinguistic sense (as a statement about the denotations, or lack of denotations, of singular terms). What is needed, according to actualism, is not that we should distinguish the concept of existence from the concept of being, but only that we should modify the way that the concept of existence (being) is represented in standard first-order predicate logic (with identity). A first-order logic of existence should allow for the possibility that some of our singular terms might fail to denote an existent object, which, according to actualism, is only to say that those singular terms are denotationless rather than what they denote are objects (beings) that do not exist. Such a logic for actualism amounts to what nowadays is called free logic." (pp. 242-243)
23. ———. 1992. "Conceptual Realism Versus Quine on Classes and Higher-Order Logic." *Synthese* no. 90:379-436.
- Contents: 0. Introduction; 1. Predication versus Membership; 2. Old versus New Foundations; 3. Concepts versus ultimate Classes; 4. Frege versus Quine on Higher-

Order Logic; 5. Conceptualism versus Nominalism as Formal Theories of predication; 6. Conceptualism Ramified versus Nominalism Ramified; 7. Constructive Conceptual Realism versus Quine's view of Conceptualism as a Ramified Theory of Classes; 8. Holistic Conceptual Realism versus Quine's Class Platonism.

Abstract: "The problematic features of Quine's 'set' theories NF and ML are a result of his replacing the higher-order predicate logic of type theory by a first-order logic of membership, and can be resolved by returning to a second-order logic of predication with nominalized predicates as abstract singular terms. We adopt a modified Fregean position called conceptual realism in which the concepts (unsaturated cognitive structures) that predicates stand for are distinguished from the extensions (or intensions) that their nominalizations denote as singular terms. We argue against Quine's view that predicate quantifiers can be given a referential interpretation only if the entities predicates stand for on such an interpretation are the same as the classes (assuming extensionality) that nominalized predicates denote as singular terms. Quine's alternative of giving predicate quantifiers only a substitutional interpretation is compared with a constructive version of conceptual realism, which with a logic of nominalized predicates is compared with Quine's description of conceptualism as a ramified theory of classes. We argue against Quine's implicit assumption that conceptualism cannot account for impredicative concept-formation and compare holistic conceptual realism with Quine's class Platonism."

"According to Quine, in one of his later works, the pioneers in modern logic, such as Frege and Russell, overestimated the kinship between membership and predication and in that way came to view set theory as logic (Quine 1970, p. 65). Such a claim, we maintain, is both false and misleading. Frege and Russell did assume a logical kinship between predication and membership, but what they meant by membership was membership in a class as the extension of a concept (where a concept is a predicable entity, i.e., a universal in the traditional sense) and not membership in a set. Sets, unlike classes, as we have said, have their being in their members, and in that regard there need be no kinship at all between predication and membership in a set. Classes in the logical sense, on the other hand, have their being in the concepts whose extensions they are, which means that any theory of membership in a class presupposes a superseding theory of predication. (3) Frege and Russell did not view set theory as logic, but they each did develop a theory of classes and they each did so based on a superseding higher-order theory of predication." (p. 382)

24. ———. 1992. "Cantor's Power-Set Theorem Versus Frege's Double-Correlation Thesis." *History and Philosophy of Logic* no. 13:179-201.  
Abstract: "Frege's thesis that second-level concepts can be correlated with first-level concepts and that the latter can be correlated with their value-ranges is in direct conflict with Cantor's power-set theorem, which is a necessary part of the iterative, but not of the logical, concept of class. Two consistent second-order logics with nominalised predicates as abstract singular terms are described in which Frege's thesis and the logical notion of a class are defended and Cantor's theorem is rejected. Cantor's theorem is not incompatible with the logical notion of class, however. Two alternative similar kinds of logics are also described in which Cantor's theorem and the logical notion of a class are retained and Frege's thesis is rejected."

"There is another problem with Russell's solution, however, in addition to that of the relativisation of classes to each logical type. This problem has to do with the fact that the particular theory of types that Russell adopted is a theory of ramified types, which, unlike the theory of simple types, is based on a constructive (i.e. 'predicative') comprehension principle. Such a constructive approach is not without merit, but it does affect the logical notion of a class in a fundamental way. In particular, because of the kind of constructive constraints imposed by the theory on the comprehension principle, Cantor's theorem, which involves objects of different

types, cannot be proved in such a framework (cf. Quine 1963, 265). That is not objectionable in itself, but it does not get at the root of the matter of the real conflict between Cantor's power-set theorem and the logical notion of class as represented by an impredicative comprehension principle.

An impredicative comprehension principle is provable in the theory of simple types. But in this framework, as in the theory of ramified types as well, Russell's paradox cannot even be stated (because of the grammatical constraints on the conditions of well-formedness), which means that the description of the class upon which Russell's paradox is based is meaningless. Thus, not only must the universal class be relativised and duplicated, potentially, infinitely many times in order to avoid Russell's paradox on this approach, but the paradox must itself be ruled as meaningless. The theory of types, whether simple or ramified, is not really a solution of the problem so much as a way of avoiding it altogether.

There is another way in which we can preserve our logical intuitions and not give up the logical notion of a class in favor of the mathematical (i.e. in favor of set theory), and yet in which not only is Cantor's theorem formulable but so is Russell's paradox—though, of course, the latter will no longer be provable. Indeed, there is not just one such way, but at least two (both of which themselves have two alternatives). On the first, it is not the logical notion of a class that must be rejected as the way of resolving Russell's paradox, but Cantor's theorem instead. This rejection is not ad hoc or arbitrary on this approach, but is based on a more general principle, which we refer to as Frege's double-correlation thesis. It is this approach that we shall turn to first. On the second and alternative approach, which we shall turn to later, the trouble lies in neither Cantor's theorem nor in the assumption that there is a universal class (both of which can be retained without contradiction on this approach), but rather in how the logic of identity is to be applied in certain contexts. On this approach, the claim that a contradiction results by combining Cantor's theorem with the assumption that the universal class exists is not a 'truism' after all but is outright false."

References

Quine, W. V. 1963 *Set theory and its logic*, Cambridge, Mass. (Harvard University Press).