

Annotated Bibliography of Nino Cocchiarella 1966 - 1985

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The Conceptual Realism of Nino Cocchiarella

BOOKS

- Cocchiarella, Nino. 1966. *Tense Logic: A Study of Temporal Reference*. Unpublished Ph.D. Dissertation, available at ProQuest Dissertation, reference number 6609326.
 University of California - Los Angeles, January 7, 1966). Committee in charge: Richard Montague, Charmain, Alfred Horn, Donald Kalish, Abraham Robinson, Robert Stockwell (VI, 251 pages).
 N.B. *the unpublished Ph. D. thesis can be ordered to ProQuest Dissertation Express*.
 Abstract: "This work is concerned with the logical analysis of topological or non-metrical temporal reference. The specific problem with which it successfully deals is a precise formalization of (first-order) quantificational tense logic wherein both an appropriate formal semantics is developed and a meta-mathematically consistent and complete axiomatization for that semantics given.
 The formalization of quantificational tense logic herein presented adheres to all the canons of logical rigor by being carried out entirely as a definitional extension of (Zermelo-Fraenkel) set theory. Model-theoretical techniques are utilized in the semantics given and the notion of a history is formally developed as the tense-logical analogue of the notion of a model for standard first-order logic with identity. Corresponding to the key semantical concept of satisfaction (and consequently of truth) in a model, by means of which the central standard notion of logical truth is defined, the notion of satisfaction (and consequently of truth) in a history at a given moment of that history is developed, from which development, in turn, the central notion of tense-logical truth is defined. An axiomatic characterization of derivability within tense logic, or t-derivability, is then presented and proved to be both consistent and complete, i.e., it is shown that an arbitrary tensed formula is tense-logically true if and only if it is t-derivable from zero premises, i.e., if and only if it is a theorem of the given axiomatic system.
 Quantification within tense logic introduces issues in no manner confronted on the sentential level. Recognition is made that quantification over objects existing prior to the time of assertion is to be distinguished from quantification over objects existing at the time of assertion, both of which in turn are to be distinguished from quantifying over objects existing at the time of assertion. Such distinct kinds of quantification are readily distinguishable within tense logic by means of incorporation of what is here called the logic of actual and possible objects. Precise semantical and syntactical formalization of this double quantification is presented prior to its use within tense logic, and completeness theorems are given for both the full system. and the restricted logic of actual objects, the latter of which may separately be taken as a formalization of a logic which can accommodate denotationless names. These several kinds of quantificational logic lead to separate

- completeness theorems stated and established for tense logic, depending on the several kinds of quantificational bases possible for this logic."
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2. Cocchiarella, Nino, Epstein, George, Dunn, J. Michael, and Shapiro, S.C., eds. 1975. *Proceedings of the 1975 International Symposium on Multiple-Valued Logic. Indiana University, Bloomington, Indiana, May 13-16, 1975*. Long Beach: IEEE Computer Society.
 3. Cocchiarella, Nino. 1986. *Logical Investigations of Predication Theory and the Problem of Universals*. Napoli: Bibliopolis.
 Table of Contents: Preface 9; Introduction 11; Chapter 1. Nominalism 29; Chapter 2. Conceptualism 65; Chapter 3. Realism 105; Chapter 4. On The Logic of Nominalized Predicates and Its Philosophical Interpretations 165; Chapter 5. Complex Predicates and The Lambda - Operator 215; Chapter 6. Two Fregean Semantics For Nominalized Complex Predicates 243-265.
 "Beginning with Aristotle's notion of a universal as that which can be predicated of things, I provide in this monograph separate logical analyses of what nominalism, conceptualism and realism take to be the predicable nature of universals. My position throughout is that such an analysis proceeds through the construction of a formal theory of predication on the one hand and a logical semantics on the other. I adopt and apply in this regard the formal and semantical techniques of my former teachers Rudolf Carnap and Richard Montague.
 One important way in which I differ from Carnap and Montague, however, is in our respective analyses of so-called "higher order" sentences - that is, sentences in which nominalized predicates, whether simple or complex, occur as the logico-grammatical subjects of other predicates. In this regard, whereas Carnap and Montague formulate and adopt one or another version of a theory of simple logical types as the framework within which to analyze such sentences, I formulate instead, relative to nominalism, conceptualism and realism, systems which do not require any grammatical type distinctions beyond those already found in standard second order predicate logic. All of the theories of predication formulated in this monograph, in other words, are second order theories, including those which contain a logic of nominalized predicates. Russell's paradox of predication, it turns out, can be resolved without resorting to a theory of types." (From the *Preface* p. 9)
 4. ———. 1987. *Logical Studies in Early Analytic Philosophy*. Columbus: Ohio University Press.

Table of Contents: Preface XI-XII; Introduction 1; Chapter 1. The Development of the Theory of Logical Types and the Notion of a Logical Subject in Russell's Early Philosophy 19; Chapter 2. Frege, Russell, and Logicism: A Logical Reconstruction 64; Chapter 3. Meinong Reconstructed versus Early Russell Reconstructed 119; Chapter 4. Frege's Double Correlation Thesis and Quine's Set Theories NF and ML 152; Chapter 5. Russell's Theory of Logical Types and the Atomistic Hierarchy of Sentences 193; Chapter 6. Logical Atomism and Modal Logic 222; Chapter 7. Logical Atomism, Nominalism, and Modal Logic 244; Index 285-293.

"The essays collected here deal with the development of analytic philosophy in the first quarter of the twentieth century. In addition to providing a historical account of early analytic philosophy, these essays also contain logical reconstructions of Frege's, Russell's, Meinong's, and Wittgenstein's views during the period in question. Several of these reconstructions can and have been used in the new logico-linguistic developments in pragmatics and intensional logic that make up the vanguard of contemporary analytic philosophy. Others, such as the interpretation of the logical modalities in logical atomism, or the determination of the objects of fiction and dreams in Meinong's theory of objects or Russell's early logic, provide a useful introduction, if not also a solution, to a number of problems confronting analytic philosophy today. Indeed, for that matter, all of the essays collected here provide a useful propaedeutic to much of the research now going on in the study of logic and language.

A number of small changes have been made in all of the essays reprinted here, mainly for stylistic purposes. Their histories are briefly indicated as follows. Chapter 1 first appeared in *Synthese*, vol. 45, no. 1 (September 1980):71—115, Copyright © 1980 by D. Reidel Publishing Company, Dordrecht, Holland. A somewhat longer version of chapter 2 first appeared in *Frege Synthesized*, L. Haaparanta and J. Hintikka (eds.), 1986, pp. 197-252, Copyright © 1986 by D. Reidel Publishing Company, Dordrecht, Holland. The present version was given as a lecture in a seminar on 13 March 1985, for the Bertrand Russell Editorial Project at McMaster University. Chapters 3 and 4 first appeared in *Journal of Philosophical Logic*, vol. 11, no. 2 (May, 1982): 183-214, and vol. 14, no. 1 (February 1985): 1-39, respectively, Copyright © 1982 by D. Reidei Publishing Company, Dordrecht, Holland. Chapter 3 was originally given as a lecture to the Société Belge de Logique et de Philosophie des Sciences, Brussels, in December 1981. Chapter 4 was my contribution to An Interdisciplinary Conference on Logic, Truth and Type Theory, given in memory of Alfred Tarski, 6-7 April 1984. Chapter 5 first appeared in *Essays in Bertrand Russell's Philosophy*, C. Wade Savage and C. Anthony Anderson (eds.), 1987, Copyright © by University of Minnesota Press, Minneapolis. Chapter 6 first appeared in *Philosophia, Philosophical Quarterly of Israel*, vol. 4, no. 1 (January 1974):41-66. It is reprinted here with the permission of the editor. Chapter 6 was given as a lecture to the Victoria Conference on Formal Ontology at the University of Victoria on 15 October 1972. Chapter 7 first appeared in *Synthese*, vol. 31, no. 1 (June 1975):23-62, Copyright © 1975 by D. Reidel Publishing Company, Dordrecht, Holland. It was originally given as a lecture to the University of North Carolina Fall Philosophy Colloquium in October 1973." (Preface, pp. XI-XII)

5. ———. 2007. *Formal Ontology and Conceptual Realism*. New York: Springer. Table of Contents: Preface XI; Introduction XIII-XXIII;
 - I. Formal Ontology.
 1. Formal ontology and conceptual realism 3; 2. Time, Being and Existence 25; 3. Logical necessity and logical atomism 59; 4. Formal theories of predication 81; 5. Formal theories of predication part II 101; 6. Intensional possible worlds 121;
 - II. Conceptual Realism.
 7. The nexus of predication 139;
 8. Medieval logic and conceptual realism 169; 9. On Geach against general reference 195; 10. Leśniewski's Ontology 215; 11. Plurals and the logic of classes

as many 235; 12. The logic of natural kinds 273; Afterword on Truth-Makers 295; Bibliography 297; Index 307-316.

"The history of philosophy is replete with different metaphysical schemes of the ontological structure of the world. These schemes have generally been described in informal, intuitive terms, and the arguments for and against them, including their consistency and adequacy as explanatory frameworks, have generally been given in even more informal terms. The goal of formal ontology is to correct for these deficiencies. By formally reconstructing an intuitive, informal ontological scheme as a formal ontology we can better determine the consistency and adequacy of that scheme; and then by comparing different reconstructed schemes with one another as formal ontologies we can better evaluate the arguments for and against them, and come to a decision as to which system it is best to adopt.

This book is divided into two parts. The first part is on formal ontology and how different informal ontological systems can be formally developed and compared with one another. An abstract set-theoretic framework, which we call comparative formal ontology, can be used for this purpose without assuming that set theory is itself a superseding ontological system. The second part of this book is on the formal construction and defense of a particular ontological scheme called conceptual realism. Conceptual realism is to be preferred to alternative formal ontologies for the reasons briefly described below, and for others as well that are given in more detail in various parts of the book. Conceptual realism, in other words, is put forward here as the best ontological system to adopt." (From the *Introduction*, p. XIII)

6. Cocchiarella, Nino, and Freund, Max A. 2008. *Modal Logic. An Introduction to its Syntax and Semantics*. New York: Oxford University Press.
- Table of Contents: 1. Introduction 1; 2. The syntax of modal sentential calculi 15; 3. Matrix semantics 45; 4. Semantics for logical necessity 61; 5. Semantics for S 5 71; 6. Relational world systems 81; 7. Quantified modal logic 119; 8. The semantics of quantified modal logic 153; 9. Second-order modal logic 183; 10. Semantics of second-order modal logic 215; Afterword 253; Bibliography 257; Index 263-268.
- "Modal logic is a theoretical field that is important not only in philosophy, where logic in general is commonly studied, but in mathematics, linguistics, and computer and information sciences as well. This book will be useful for students, researchers, and professionals in all of these and related disciplines. The only requirement is some familiarity with first-order logic and elementary set-theory.
- The main outline of this book is a development of the logical syntax and semantics of modal logic in three stages of increasing logical complexity. The first stage is a comprehensive development of sentential modal logic, which is followed by a similarly comprehensive development of first-order modal predicate logic. The final stage is a development of second-order modal predicate logic. These stages are introduced gradually, with increasing difficulty at each stage. Most of the important results in modal logic are described and proved in each of their respective stages. This book is based on a series of lectures given over a number of years at Indiana University by the first author. A draft of the book has also been used by the second author in Costa Rica and Mexico. The book is organized as follows.
- We begin in chapter 1 with concatenation theory and the logistic method. By means of this theory and method we describe the construction of expressions, formal languages, and formal systems or calculi. Different modal calculi are then constructed in chapter 2. These cover all of the well-known systems, S1–S5, of Lewis and Langford's 1932 classic *Symbolic Logic*. As already indicated, these systems are constructed first on the level of sentential (or propositional) logic and then later in chapter 7 on the level of first-order predicate logic, where we distinguish the quantified modal logic of actualism from that of possibilism. The systems are then extended yet again to the level of second-order modal predicate logic in chapter 9, where the notion of existence that is central to the actualism-possibilism distinction is given a deeper and finer-grained analysis in

terms of existence-entailing concepts, as opposed to concepts that do not entail existence." (From the *Introduction*, p. 1)

ESSAYS

1. Cocchiarella, Nino. 1966. "A Logic of Actual and Possible Objects." *Journal of Symbolic Logic* no. 31:688.
See note in 1966c.
2. ———. 1966. "A Completeness Theorem for Tense Logic." *Journal of Symbolic Logic* no. 31:689.
See note in 1966c.
3. ———. 1966. "Modality within Tense Logic." *Journal of Symbolic Logic* no. 31:690.
Note to the reprint of these three items in Karel Lambert (ed.) *Philosophical Applications of Free Logic*, New York: Oxford University Press 1991): "The abstracts are summaries of lectures given at the December, 1965 meetings of the Association for Symbolic Logic.
(A preliminary version of those lectures was given at UCLA in 1963, and a final version was given at UCLA in the spring of 1965 at a public lecture constituting the defense of my doctoral dissertation.)"
4. ———. 1968. "Some Remarks on Second Order Logic with Existence Attributes." *Noûs* no. 2:163-175.
"In *Past, Present and Future* A. N. Prior has suggested an approach towards the concept of existence where, following medieval logicians, we are to distinguish "between predicates (like 'is red', 'is hard', etc.) which entail existence, and predicates (like 'is thought to be red', 'is thought of', etc.) which do not" (p. 161). Let us refer to attributes (including relational attributes) which are designated by the former kind of predicate as existence attributes, or for brevity, e-attributes. It is suggested then that "x exists" is to be defined as "there is some e-attribute which x possesses". A formalization of this (at least) second order logic of existence was recently brought about and reported on by the present author in [6]. The formalization was shown to be complete in the sense corresponding to the completeness of standard second order logic, i.e. in the sense which encompasses normal, non-standard as well standard models. (Cfr. A. Church, *Introduction to Mathematical Logic* (1956) - § 54). I should like in the present paper to discuss some of the philosophical issues involved in this formalization as well as some issues concerning the general notion of e-attribute." (p. 163)
5. ———. 1969. "A Substitution Free Axiom Set for Second Order Logic." *Notre Dame Journal of Formal Logic* no. 10:18-30.
"In what follows we present an adequate formulation of second order logic by means of an axiom set whose characterization does not require the notion of proper substitution either of a term for an individual variable or of a formula for a predicate variable. The axiom set is adequate in the sense of being equivalent to standard formulations of second order logic, e.g., that of Church [1]. It is clear and need not be shown here that every theorem of the present formulation is a theorem of the formulation given by Church. It of course will be shown here, however, that each of Church's axioms are theorems of the present system and that each of his primitive inference rules is either a primitive (and only modus ponens is taken as a primitive rule here) or a derived rule of the present system.
The importance of obtaining an axiomatic formulation such as herein described lies partly in the significance of reducing any axiom set to an equivalent one which involves fewer metalogical notions, especially such a one as proper substitution. However, of somewhat greater importance, it is highly desirable that we possess a

formulation of both first and second order logic which can be extended without qualification to such areas as tense, epistemic, deontic, modal and logics of the like. Now proper substitution especially has been the main obstacle to such unqualified extensions of standard logic, and we take it to be of no little significance that at least for first order logic (with identity) a substitution free axiomatic formulation has been provided. (1) The present system extends this earlier result to the level of second order logic. (2)

A second difficulty in unqualified extensions of standard logic concerns the form which Leibniz' law, i.e., the law regarding interchangeability *salva veritate*, is to take. Generally, in the extensions of standard logic to modal logic, this law has been formulated in an unqualified form applicable to all contexts, thereby lending credence to the questionable view that only "intensions" or the like can serve adequately as values of the variables for such systems. In the substitution free formulations of first order logic cited, however, Leibniz' law is axiomatically formulated only for atomic contexts, and the qualified form or forms the law takes for contexts involving non-standard formula operators is given in the statement of metatheorems. (3) But again, it is a far different matter having such qualifications stipulated in the form of metatheorems as opposed to having them built directly into the characterization of the logical axioms. As we have said, it is desirable that the standard logical axioms for either first or second order logic be so that axiomatic extensions of standard logic can be made without qualification.⁴ This desirable feature of the substitution free formulations of first order logic mentioned is retained in our present second order system." (pp. 18-19)

(1) Such a formulation is given by A. Tarski in [2] and developed by D. Kalish and R. Montague in [3]. The present author in [4] and [5] has also formulated a substitution free axiomatization of first order logic without "existential presuppositions."

(2) Of course, when extending either first or second order logic to tense, epistemic, deontic, or modal logic, qualifications in metatheorems regarding principles of proper substitution will be required. Nevertheless, it is a far different matter having such qualifications stipulated in the form of metatheorems than it is having them built directly into the characterization of the logical axioms themselves.

(3) cf. [4], lemma 4.27 (p. 108) and the discussion on page 106. The objections against an unqualified, general version of Leibniz' principle (or interchangeability *salva veritate*) are applicable when certain special 'Opaque' contexts are involved, be they modal or otherwise. But all such contexts are—or should be when properly formalized—other than atomic, their "opacity" being generated within the scope of special formula operators. Atomic formulas, because they are atomic, will contain no occurrences of such operators and consequently will uphold par excellence the Leibnizian principle unqualifiedly."

References

[1] Church, A., *Introduction to Mathematical Logic*, Vol. I, Princeton, 1956.

[2] Tarski, A., "A Simplified Formalization of Predicate Logic with Identity," *Arch. f. Math. Logik u. Grundl.*, 7 (1965), 61-79.

[3] Kalish, D., and Montague, R., 'On Tarski's Formalization of Predicate Logic with Identity,' *Arch. f. Math. Logik u. Grundl.*, 7 (1965), 81-101.

[4] Cocchiarella, N., *Tense Logic: A Study of Temporal Reference*, Doctoral Thesis, UCLA, 1966.

[5] Cocchiarella, N., "A Logic of Actual and Possible Objects," *The Journal of Symbolic Logic*, Vol. 31 (1966), 688f.

6. ———. 1969. "A Second Order Logic of Existence." *Journal of Symbolic Logic* no. 34:57-69.

"A. N. Prior in [9] has suggested an approach towards a second order logic of existence where, following medieval logicians, we distinguish "between predicates (like 'is red', 'is hard', etc.) which entail existence, and predicates (like 'is thought to be red', 'is thought of', etc.) which do not." (2) Let us refer to attributes (including relational attributes) which are designated by the former kind of

predicate as existence attributes, or for brevity, e-attributes. It is suggested then that '*x exists' be defined as 'there is some e-attribute which x possesses'. In what follows, this approach regarding the concept of existence is formalized semantically as well as syntactically, and a completeness theorem is established corresponding to the completeness (in a secondary sense, i.e., as including normal, nonstandard models) of standard second order logic (as formulated, for example, in Church [1])."

[For a more philosophical discussion of the present system, especially of the substitution free form of its axiom set, cf. *Some Remarks on Second Order Logic with Existence Attributes*].

(2) p. 161

References

[1] A. Church, *Introduction to mathematical logic*, vol. I, Princeton, N.J., 1956.

[9] A. N. Prior, *Past, present and future*, Oxford Univ. Press, Oxford, 1967.

7. ———. 1969. "Existence Entailing Attributes, Modes of Copulation, and Modes of Being in Second Order Logic." *Noûs* no. 3:33-48.

"Recently, in [5], I formulated a second order logic of existence which centered around the distinction between those attributes that entail existence and those that do not. (1) The distinction provides

an especially apt explication of the concept of existence and is for this reason especially pertinent to pragmatics and intensional logic, encompassing as they do such areas as tense, epistemic, deontic and modal logic. (2) For example, apropos of tense logic some attributes, such as being red, being round, being hard, etc., cannot be possessed at a time except by objects existing at that time. Other attributes, especially relational attributes between objects whose "lifespans" need not overlap, such as being an ancestor of everyone (someone) now existing, being remembered by someone now existing, (3) etc., may very well be possessed by objects which no longer exist; others, e.g., being a future descendant of everyone (someone) now existing, may be possessed by objects which have yet to exist. Still other attributes such as being believed to be a flying horse may be possessed by objects which never exist. Those attributes which entail existence (at the time of their possession) I shall call existence attributes, or for brevity, e-attributes. By a relational e-attribute I mean an attribute which entails existence with respect to each of its argument places.

In the present paper I shall discuss some of the motivation for distinguishing e-attributes from attributes in general. As indicated, this motivation depends essentially on the desire to use logistic systems in which we are allowed to recognize modes of being other than that of actual existence, e.g., such modes in tense logic as past and future existence, or, in the logic of belief, the mode of "intentional inexistence". As also indicated, the concept of existence is central to this discussion and I shall here examine informally its explication in terms of e-attributes. In a sequel to the present paper I shall present and discuss a formal analysis of this explication in the context of a semantics for standard second order logic, with quantification over e-attributes distinguished from quantification over attributes in general. The focal point of the formal discussion will be the issue of the logical priority of the notion of e-attribution over that of being an existing object, a priority which exemplifies that of the intensional over the extensional and which, for its clarification, requires some observations on the class-attribute distinction." (pp. 33-34)

(1) For a more philosophical discussion of the axiom set for this formalization, see [4]. I follow Carnap [1], p. 5, in using attribute' to comprehend both properties and relations (in-intension). Properties are 1-place or unary attributes and are designated by 1-place predicate expressions. Extending Carnap's terminology, propositions are understood to be designated by 0-place predicate expressions and are therefore construed as 0-place attributes (whose extensions are truth-values).

(2) Cf. R. M. Montague [9] and [10] for an elegant and philosophically stimulating formulation of pragmatics and intensional logic. Montague's formulation of

intensional logic, supplemented by the distinction between existence entailing and other kinds of attributes, is perhaps the most appropriate general logical framework to which the discussion and observations of the present paper lead.

(3) This example is given by R. M. Montague in [9] and [10].

References

[1] Carnap, R., *Introduction to Symbolic Logic and Its Applications* (New York: Dover Press, 1958).

[4] Cocchiarella, N., "Some Remarks on Second Order Logic with Existence Attributes," *Noûs*, II, 2 (1968): 165-175.

[5] Cocchiarella, N., "A Second Order Logic of Existence," *JSL*, forthcoming [1969].

[9] Montague, R. M., "Pragmatics and Intensional Logic," forthcoming in *Dialectica*. [published in *Synthese*, Vol. 22, No. 1/2, Semantics of Natural Language, II (Dec., 1970), pp. 68-94]

[10] Montague, R. M., "Pragmatics," in Raymond Klibansky (ed.), *Contemporary Philosophy—La Philosophie Contemporaine* (Florence: 1968).

8. ———. 1969. "A Completeness Theorem in Second Order Modal Logic."

Theoria. A Swedish Journal of Philosophy no. 35:81-103.

"In what follows we present a second order formulation of S5 which is shown to be complete relative to a secondary sense of validity corresponding to that relative to which standard second order logic is known to be complete.(1) In our semantical metalanguage we consider various indexed sets of possible worlds and allow that not all objects existing in one indexed world need exist in another. However, as we have therefore confessed in the metalanguage our ontological commitment to all the objects that exist in one world or another, we acknowledge and formalize this confession in our object languages through allowing for quantification over *possibilia*.

Our means for distinguishing the existent from the mere possible is through a distinction between those attributes that entail existence (with respect to each of their argument places), referred to hereafter as e-attributes, and those attributes that do not.² Accordingly, we understand 'x exists' to mean 'There is some e-attribute which x possesses', thus rendering existence essentially impredicative. An alternative and equivalent route—but which we shall not follow here—is possible through taking existence as primitive in the form of quantification over existing objects and defining e-attributes as those attributes which necessarily are possessed only by existing objects." (p. 81)

(1) Cf. Henkin [9].

(2) For a modal free complete (in a secondary sense) formulation of second order logic with existence attributes, see Cocchiarella [5]. For a more philosophical discussion of this approach toward existence, see Cocchiarella [6] and [7].

References

[1] Church, A., *Introduction to Mathematical Logic*, Vol. I, Princeton, 1956.

[5] Cocchiarella, N., "A Second Order Logic of Existence", *Journal of Symbolic Logic*, vol. 34 (1969).

[6] Cocchiarella, N., "Some Remarks on Second Order Logic with Existence Attributes", *Noûs*, 2, 1968, 165-175.

[7] Cocchiarella, N., "Existence Entailing Attributes, Modes of Copulation and Modes of Being in Second Order Logic", *Noûs*, 3, 1969, 33-48.

[9] Henkin, L., "Completeness in the Theory of Types", *Journal of Symbolic Logic*, vol. 15 (1950), 81-91.

9. ———. 1972. "Properties as Individuals in Formal Ontology." *Noûs* no. 6:165-187.

"Russell's supposed paradox of predication has occasionally been cited as a source for lessons in ontology. So, for example, Grossmann in [6] has argued that one of the lessons of Russell's paradox is that there are no complex properties. A recent re-evaluation of the supposed paradox, however, has led me to the conclusion that there is no paradox (cf. [3]). And of course where there is no paradox, there are no lessons of paradox.

There may, however, be lessons of non-paradox, especially if instead of contradiction what results is a highly instructive ontological oddity. In what follows I shall briefly review the considerations that led me to conclude that there is no paradox but instead only this ontological oddity with instructive lessons of its own, relative of course to the ontological framework within which it occurs. I shall then briefly consider several ways of responding to this oddity, where each response presupposes an alternative ontological framework relative to which the response accounts for the oddity by either showing it to rest on an ontological error, as with Grossmann's response, or by mitigating its effect through what purports to be a deeper or wider framework than the original one in which the oddity occurs." (p. 165)

References

[3] Cocchiarella, N., "Whither Russell's Paradox of Predication," forthcoming in *Logic and Ontology*, vol. 2 of *Studies in Contemporary Philosophy*, edited by M. K. Munitz and to be published by New York University Press [1973].

[6] Grossmann, R., "Russell's Paradox and Complex Properties," *Noûs*, Vol. 6, No. 2 (May, 1972), pp. 153-164.

10. ———. 1973. "Whither Russell's Paradox of Predication?" In *Logic and Ontology*, edited by Munitz, Milton K., 133-158. New York University Press.

Contributions to a seminar on ontology held under the auspices of the New York University Institute of Philosophy for the year 1970-1971.

"Russell's paradox has two forms or versions, one in regard to the class of all classes that are not members of themselves, the other in regard to "the predicate: to be a predicate that cannot be predicated of itself."(1) The first version is formulable in the ideography of Frege's *Grundgesetze der Arithmetik* and shows this system to be inconsistent. The second version, however, is not formulable in this ideography, as Frege himself pointed out in his reply to Russell. (2) Nevertheless, it is essentially the second version of his paradox that leads Russell to avoid it (and others of its ilk) through his theory of types.

The first version is of course the relevant version with respect to any formulation of the theory of types in which membership in a class is the fundamental notion, that is, a formulation utilizing 'ε' as a primitive binary predicate constant.(3) However, Russell's theory of types (even ignoring its ramification) is essentially concerned with the notion of predication, and only indirectly through the (philosophically questionable) interpretation of predication as the membership relation is the first version of his paradox relevant to this formulation.

Apparently, Russell saw his paradox as generating an aporetic situation in regard to two fundamental "notions," namely, the notion of membership (in a class) and the notion of predication (of an attribute).(4) In regard to the notion of membership, the application of Russell's paradox is not here brought into question. However, in regard to the notion of predication, the applicability of the reasoning grounding Russell's paradox will here be very much brought into question. Indeed, I shall claim that in this case the paradox fails.(5)" (pp. 133-135)

(1) "Letter to Frege," reprinted in [10], p. 125.

(2) "Letter to Russell," *ibid.*, p. 128.

(3) Cf. [5], p. 140 for a specific formulation of this kind of type theory.

(4) Gödel (cf. [6], p. 131f.) distinguishes these two forms of Russell's paradox by referring to them as the "extensional" and the "intensional" forms, respectively. For the purposes of the present paper, this distinction is preferable to Ramsey's different but better known distinction between "logical" and "semantical" paradoxes.

(5) With this failure of course goes a primary if not sole motivation for the simple theory of ontological types of third and higher order. The ontological scheme of second-order logic remains unaffected, having as it does a natural motivation of its own. Ramification also has its own motivation, and it may be appended to second-order logic (cf. [2], §58.) even though historically it was first appended to the simple theory of types.

References

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11. ———. 1974. "Fregean Semantics for a Realist Ontology." *Notre Dame Journal of Formal Logic* no. 15:552-568.
- "T* is a logicistic system¹ designed to represent the original ontological context behind Russell's paradox of predication. It encompasses standard second order logic, hereafter referred to as T, but goes beyond it by allowing predicate variables to occupy subject positions in its formulas.
- Because of a violation of the restrictions imposed for the proper substitution of a formula for a predicate variable, Russell's argument fails in T*. Indeed, not only is T* consistent but it is also a conservative extension of T.
- It has been suggested that one way of understanding this result is to construe occurrences of predicates in subject positions as referring, not to the properties which occurrences of the same predicates in predicate positions designate, but instead, to individual objects associated with these properties.⁴ Such a suggestion of course is reminiscent of Frege's ontology. And were it not that Frege is quite insistent in viewing predicates as "unsaturated" expressions and therefore not qualified as substituends for subject positions which can be occupied only by "saturated" expressions, it might be tempting to construe T* as representative of Frege's ontology. Be that as it may, the disproof of the principle that indiscernible properties are co-extensive, which is all that Russell's paradox comes to in T*, is reinterpreted according to this suggestion so as to show merely a variant of Cantor's theorem. And that after all is rather appropriate, since Russell's argument for his supposed paradox is really but a variant of Cantor's argument for his theorem.
- In what follows we formulate the suggestion semantically and show that although the semantics thus provided does not characterize T*, it does characterize a certain rather interesting subsystem T** of T* supplemented by the extensionality principle that co-extensive properties are indiscernible.⁽⁵⁾ The supplemented system, T**+(Ext*), no doubt appears bizarre from the point of view of the original ontological background represented by T*—since in this ontology not all indiscernible properties are co-extensive whereas, according to the supplement, all co-extensive properties are indiscernible, thus suggesting co-extensiveness to be a stronger connection between properties than is the indiscernibility relation.
- On the other hand, from the point of view of its quasi-Fregean semantics, the supplement seems rather natural—for according to this semantics the supplement amounts to the stipulation that the same individual object is to be associated with co-extensive properties. Fregean naturalness aside, it should perhaps be noted that the existence of a model-set-theoretic semantics characteristic for T*—or of T* supplemented with principles natural to the ontology of T*—remains yet an open problem." (pp. 552-553)
- (2) Cf. [2], §6.
- (3) *Ibid.*, §5. We should avoid using 'identical' in place of 'indiscernible' here. In [3], Meyer has shown that according to T* there exists no relation which satisfies full substitutivity, and, accordingly, insofar as full substitutivity is taken to be a necessary feature of identity, there is and can be no identity relation in the ontology of T*.
- (4) This suggestion is implicit, though only in a partial way, in the argument independently arrived at by Zorn and Meyer that T* is a conservative extension of T. It is explicit in the type of model defined below as quasi-Fregean and first recommended to the author as characteristic of T* by N. Belnap.

(5) It is easily seen from the proof in [2] that T^* is a conservative extension of T , that this extensionality principle is not a theorem of T^* —nor for that matter is its negation.

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12. ———. 1974. "Formal Ontology and the Foundations of Mathematics." In *Bertrand Russell's Philosophy*, edited by Nakhnikian, George, 29-46. London: Duckworth.
- "In his paper, 'The Undefinability of the Set of Natural Numbers in the Ramified *Principia*', [*] Myhill has shown that the general concept of a natural number or finite cardinal - general enough, that is, to yield the induction schema - is not definable in terms of ramified type theory in essentially its original form and without the axiom of reducibility. In my commentary I shall examine Myhill's concluding philosophical remarks within the context of general metaphysics or what below I call formal ontology. I shall especially be concerned with the sense in which ramified type theory (without the axiom of reducibility) purports to represent a constructive philosophy of mathematics. In addition, I shall sketch several forms of realism according to which the claim that "impredicativity is present in mathematics from the beginning" is true in an especially apt and interesting sense that goes beyond that intended by Myhill." (p. 29)
- [*] In George Nakhnikian (ed.), *Bertrand Russell's Philosophy*, pp. 19-27.
13. ———. 1974. "Logical Atomism and Modal Logic." *Philosophia. Philosophical Quarterly of Israel* no. 4:41-66.
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- "Logical atomism has been construed as both a realist and a nominalist ontology. Despite their different ontological commitments, proponents of both types of atomism have tended to agree that modal operators for necessity and possibility, and thereby strict entailment too, are totally alien to the ontological grammar of logical atomism. The reason for this, apparently, is that any inclusion of modal operators in the ontological grammar of logical atomism, whether that grammar be of the nominalist or realist variants, would represent a commitment to internal properties and relations with material content. And in logical atomism, of course, all internal properties and relations, be they of objects or of situations, are formal and not material in nature. (Cf. Wittgenstein, *Tractatus Logico-Philosophicus*, ([TR]), 4.122).
- However, to the contrary, we shall argue that not only are propositional connectives for logical necessity and possibility, and thereby strict entailment too, no more alien to the ontological grammar of logical atomism than are connectives for conjunction and disjunction, but, moreover, that the formal or internal properties and relations of objects and situations in the ontology cannot be adequately represented by the propositional forms of that grammar unless connectives for logical necessity and possibility are included (or definable by others so included) therein.
- That is, we shall argue that connectives for logical necessity and possibility, together with their proper "logico-syntactical employment" ([TR], 3.327), represent formal, and not material, internal "properties," and, moreover, that these formal, internal "properties" are part of the ontology of logical atomism and cannot be adequately represented without the inclusion of such connectives in the ontological grammar of any formal system purporting to represent that ontology.(1)
- Our position and argument, incidentally, applies only to modal operators for logical necessity and possibility. All other modal operators, we agree, because they purport to represent internal "properties" or "relations" with real material content (e.g., causality, and even temporality via tense logic), are strictly prohibited within the metaphysical framework of logical atomism. "Superstition is nothing but belief in

the causal nexus" ([TR], 5.1361). "The only necessity that exists is logical necessity" ([TR], 6.37).

Moreover, our concern here shall be with logical atomism as the metaphysical framework for a type of formal ontology. Our concern will not be with logical atomism as the framework for either a theory of meaning or a theory of knowledge. Accordingly, neither the Carnapian theory of *Protokolsätze* nor the Tractarian picture theory of meaning are essential to our present purely ontological considerations. We should note, however, that the Tractarian theory of elementary propositions as pictures contains both a theory of predication and a theory of meaning. It is the theory of predication that is an essential part of the ontology of logical atomism.

In the present chapter we shall limit our formal developments to the level of analysis dealing solely with propositional connectives. Our next chapter will deal with nominalist logical atomism where only individual variables are bindable but where atomism's theory of predication enters the ontological grammar in a fundamental way. That chapter will also contain a description of several variants of realist logical atomism, one in which material properties and relations of objects are themselves objects, and another where material properties and relations of objects, though indicated by bound predicate variables (as in the first variant of realism), are not themselves objects (values of individual variables) but are nexuses or modes of configurations of objects (as they are in nominalism where they are not indicated by bound predicate variables)." (pp. 222-223 of the reprint)

14. ———. 1974. "A New Formulation of Predicative Second Order Logic." *Logique et Analyse* no. 65-66:61-87.

"In what follows, a predicative second order logic is formulated and shown to be complete with respect to the proposed model theoretic semantics. The logic differs in certain fundamental ways from the system formulated by Church in [1] § 58. The more important differences are noted and discussed throughout the present paper. A more specialized motivation for the new formulation is outlined in § 2.

In regard to the motivation for Church's formulation, this will be found in its natural extension to ramified type theory (without the axiom of reducibility). Within this larger framework, the theory of predication represented by such a formulation can be seen to be constructive: higher order entities are constructible from entities of lower order, with real, non-constructed individuals as the entities of lowest order. Set theory, to whatever extent it is representable in the framework, appears in the ramified hierarchy only after propositional functions are allowed to be arguments of third and higher order predicates. To introduce sets as real, non-constructed individuals of lowest order would be antithetical to the framework's constructive theory of predication and in violation of its philosophical motivation."

(...)

"Essential to this proposal, however, is a view of the predicative/impredicative distinction radically different from that found in ramified type theory — and hence in Church's formulation of predicative second order logic. The latter framework (barring the axiom of reducibility) represents a constructive theory of predication that rules out all manner of categorial content (indicated by bound predicate variables) or logistic efficacy for impredicative contexts. In the proposed, modified Fregean theory, however, impredicative contexts (wffs) are allowed to have logistic efficacy — and perhaps even categorial content if standard quantifiers ranging over all properties and relations are retained as well.

If, on the one hand, only quantifiers for predicatively specifiable properties and relations are allowed, then in this new formulation of predicative second order logic impredicative contexts — which in general will contain free (schematic) predicate variables or certain predicate constants — will be syncategorematic expressions, since they will not then be permissible substituends of generalized predicate variables. This does not mean that they must then be accorded null content. They may instead represent logical or formal content variant to what Frege calls second and third level "concepts". (4)

This logical content would in effect be the basis of their logistic efficacy. (5) If, on the other hand, these impredicative contexts are to be given categorial content by retaining standard quantifiers, then care must be made to distinguish these quantifiers from those ranging over only predicatively specifiable properties and relations. Both kinds of quantifiers will bind the same variables, but impredicative wffs will be permissible substituends only of variables bound by the one quantifier.

(6)

They remain impermissible substituends of variables bound by the quantifiers for predicative properties and relations.

In the system to be formulated here we are concerned only with the first of the above alternatives, although once formulated it is easily extended to the richer framework. (7)" (pp. 61-64)

(4) We should distinguish at least two kinds of content that expressions of a formal system might have. The first is generally called descriptive, but historically has been called categorial, which we prefer here since even without (applied) descriptive constants the content is still indicated by bound variables, (Hence our reference to categorial content.) The second is generally called logical or formal, or, traditionally, syncategorematic and is understood to be immanent to the logistic system in question. This latter content is usually said to be null or non-existent because it is not denoted or designated by corresponding constants, or, equivalently, because it is not indicated by any type of bound variable. (It may however be «indicated» in a secondary sense by free or schematic variables, and therefore also by constants that are substituends of these free or schematic variables.)

This rather standard view is untenable, however; for if the corresponding or associated expressions have logistic efficacy in the system, that fact can be accounted for only in terms of their representing content of some sort.

On the other hand, because of its immanency, this content need not be therefore accorded categorial existence, i.e., it need not be indicated by bound variables. Our point here, however, is that categorial existence is not the only philosophically viable notion of existence. In ramified type theory (without the axiom of reducibility), impredicativity has neither categorial nor syncategorial existence. In the new predicative second order logic, impredicativity has syncategorial but not categorial existence. In standard second order logic, impredicativity has categorial existence.

(5) A perspicuous representation of this logistic efficacy is the rule (S) of substitution of wffs for free (schematic) predicate variables or constants occurring in theorems. (Cf. 4 below for a description and derivation of (S).) This rule, though derivable in the predicative second order logic formulated here, is not derivable in Church formulation. Indeed, its addition there as a new rule results in standard, and not predicative, second order logic. This is not the case in the new formulation given here.

(6) The principle of universal instantiation, (UI), of wffs those containing as well as those not containing bound predicate variables — for a generalized predicate variable is now both formulable and valid when the generalized predicate variable is bound by the standard quantifier. This principle implies the weaker rule (S) and therefore contains, and goes beyond, the logistic efficacy of that rule. (7)" (pp. 61-64)

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15. ———. 1975. "On the Primary and Secondary Semantics of Logical Necessity." *Journal of Philosophical Logic* no. 4:13-27.

"The semantical development of modal logic over the past fifteen years has incorporated a particular model-theoretic artifice which has received little or no critical attention. It is our contention that this artifice introduces, at least within conceptual frameworks typified by logical atomism, a subtle form of descriptive as opposed to merely formal content into the semantics of modal operators. This is

particularly noteworthy at least for systems containing operators for the so-called logical modalities, e.g., logical necessity or possibility, or their cognate binary modality, strict implication; for, if any modal operators or connectives had ever been conceptually ordained to represent merely logical or formal operations with no material or descriptive content, it is such as these. Yet, as a result of this model-theoretic artifice, that is precisely what they fail to do.

Relative to a given non-empty universe of objects and a set of predicates of arbitrary (finite) addicity (representing the nexuses of atomic or basic states of affairs), the artifice in question concerns allowing modal operators to range (in their semantical clauses) over arbitrary non-empty subsets of the set of all the possible worlds (models) based upon the given universe of objects and the set of predicates in question. The intuitive and natural interpretation of modal operators for logical modalities, however, is that they range over all the possible worlds (models) of a logical space (as determined by a universe of objects and a set of predicate-nexuses) and not some arbitrary non-empty subset of that totality. The latter interpretation, by allowing the exclusion of some of the worlds (models) of a logical space, imports material conditions into the semantics of modal operators. This exclusion, however appropriate for the representation of non-logical (e.g., causal or temporal) modalities, is quite inappropriate for the representation of what are purported to be merely formal or logical modalities.

This model-theoretic artifice of allowing the exclusion of some of the worlds (models) of a logical Space goes back to Kripke [5] where the notion of universal validity is used instead of the intuitive and primary notion of logical truth. Later semantical developments, by Kripke and others, retained the artifice and supplemented it with additional model-theoretic features, e.g., special accessibility relations between the non-excluded worlds, or semantical clauses allowing objectual quantifiers to range over arbitrary subsets of the universe of objects (thereby importing material content into the semantics of these operators as well). Such additions only deepened and supplemented the type and variability of the material content already induced by modal operators as a result of the artifice in its simplest form. And in that regard, however appropriate these additions may be for the representation of particular non-logical modalities, they only mark a further departure from the supposed purely formal content of operators for logical modalities. For this reason we shall ignore these later developments here and restrict our observations to some of the implications of the artifice in its original and simplest form. It should be kept in mind, however, that our discussion pertains only to operators for the so-called logical modalities." (pp. 13-14)

(...)

"Concluding remarks.

It is not our contention that we should eschew either the model-theoretic artifice of allowing modal operators to range over only some and not all of the worlds (models) of a given logical space or the related artifice of allowing («-place) predicate quantifiers to range over only some and not all of the sets (of «-tuples) of objects in the universe of that space. Indeed, we agree that such artifices are quite appropriate and may in fact be required for operators purportedly representing non-logical modalities (e.g., temporal or causal modalities) or for quantifiers which purportedly range over attributes of a restricted form of content (e.g., perceptual content, or existence-entailing content where past- and future-existence are distinguished from present-existence).

It is our contention, however, that the employment of such an artifice is inappropriate in the semantics of what one considers to be a purely formal or syncategorematic sign. The fact that a secondary semantics which includes such an artifice yields a proof of completeness where the primary semantics showed incompleteness instead does not itself justify employment of the artifice. Rather, to adopt a secondary semantics for this sort of reason is, in our view, to already call into question the sense in which the sign is said to be syncategorematic or the sense in which the content purportedly represented is said to be of a purely formal nature. That of course may in the end be the appropriate question to raise in regard to all

our so-called syncategorematic or logical constants. But to raise the question and to answer it adequately are two entirely different enterprises." (p. 28)

References

[5] Kripke, S., 'A Completeness Theorem in Modal Logic', *Journal of Symbolic Logic* 24 (1959), 1-14.

16. ———. 1975. "Second Order Theories of Predication: Old and New Foundations." *Noûs* no. 9:33-53.

"Second-order theories of predication are based on the assumption that a semantical or ontological interpretation of the forms of predication found in first-order languages will be philosophically adequate only if within the framework of the interpretation there exist entities corresponding to (some if not all of) the predicates occurring in these forms. These entities, depending on the theory in question, may or may not be projected as existing in reality independently of the structure of thought. For convenience, however, we shall refer to them in either case as properties when they are projected as corresponding to monadic predicates and as n-ary relations when they are projected as corresponding to n-place predicates, for $n > 1$.

Now the nature of the correspondence in which properties and relations are purported to stand to predicates in second-order theories is such that it cannot be identified with or reduced to the relation of denotation between singular terms, e.g., individual constants or variables, and the individuals or objects which they are understood to denote. In some second-order theories it cannot be understood as a relation at all, though in others it will (properly) include the singular-term denotation relation (in the sense that properties and relations can also be denoted therein by singular terms) while still going beyond it in ways that are peculiar to predicates. For this reason, quantification over the theoretically projected or posited properties and relations is primarily effected through quantified predicate variables and not, as it were, through a form of restricted quantification over one or another kind of individual. Informally, we say that properties and relations have in this regard a *predicative nature*, though in some theories they may have a *nominative nature* as well.

In what follows we shall be concerned, though somewhat unevenly, with this distinction between second-order theories in which properties and relations have only a predicative nature as opposed to those in which they are purported to have a nominative nature as well. The two general types of second-order theory we have in mind, then, are distinguished according to (1) whether the nature of the correspondence between predicates on the one hand and properties and relations on the other is to (properly) include the singular-term denotation relation so that predicates, within the framework of the theory, are allowable substituends of individual variables; or (2) whether the purported mode of being of properties and relations is strictly of a predicative nature which excludes their being arguments or logical subjects of predication in any sense which is logically similar to that in which individuals in general are. In the first type of theory, properties and relations are themselves individuals, i.e., have a nominative as well as a predicative nature, whereas in the second the categories or modes of being purportedly indicated by quantified predicate and individual variables are ontologically disjoint. Following Frege, we shall speak of properties and relations as *unsaturated* entities when they are projected entities of a theory of the latter sort." (pp. 33-34)

17. ———. 1975. "Logical Atomism, Nominalism, and Modal Logic." *Synthese* no. 31:23-62.

Reprinted as Chapter 7 in *Logical Studies in Early Analytic Philosophy*, pp. 244-275.

"Logical atomism, through its theory of logical form, provides one of the most coherent formal ontologies in the history of philosophy. It is a coherence which, whether we agree with the ontology or not, renders the framework important and useful as a paradigm by which to compare and better evaluate the coherence of

alternative systems based upon alternative theories of logical form and especially alternative theories of predication.

As the basis of a formal ontology, logical atomism, aside from the differences between its realist and nominalist variants, specifies not only a 'deep structure' ontological grammar within which all analysis must ultimately be resolved, but determines as well a logistic for that grammar. Both together constitute the formal ontology and serve to indicate how logical atomism views the fundamental structure of reality. Thus, for example, the grammar serves to indicate the formal as well as the material categories of being acknowledged by the ontology, while the logistic, by regulating the proper 'logico-syntactical employment' ([TR], 3.327) of the expressions of that grammar serves to indicate not only the logical 'scaffolding of the world' ([TR], 6.124) but supplements the grammar in its presentation of the ontological structure of reality.

The distinction between logical scaffolding and ontological structure is fundamental to atomism and pertains to a distinction between material and formal content that grammar alone is insufficient to represent. It is a distinction that any proposed formalization of logical atomism must account for (through the Doctrine of Showing) in order to be an adequate formal representative of that ontology. It is a distinction, however, or so it will be argued here, that cannot be made without the introduction of modal operators for logical necessity and possibility.

The argument for this last claim was already given in chapter 6, but it was there restricted to the level of logical analysis dealing solely with propositional connectives."

(...)

"In what follows we shall be concerned with the problematic extension of these results to the level of analysis involving quantifiers for objects as concrete particulars along with some means for expressing their self-identity and mutual difference. On this level, logical atomism's theory of predication enters our considerations in a fundamental way. For according to that theory, only elementary predications represent or 'picture' a structure with material content, and that content is in all cases external to the constituents of the structure. Such a structure is an atomic situation (*Sachlage*) and the externality of its content to its constituents consists in both it and its complement being logically possible. The difficulty here is that since objects are quantified over, they are part of the world and therefore contribute to the ontological content of the world (cf. [TR] 5.5561); and in that regard their self-identity and mutual difference or nonidentity, and thereby their total number, would *prima facie* seem to involve material content. Yet, in atomism, an object's self-identity or nonidentity with any other object is not an external condition of that object, (3) and, as a consequence of the dependence of logical space on reality, it is logically impossible for the totality of objects, no less the number of that totality, to differ from world to world. In other words, in logical atomism, if not in other ontologies, identity and difference, as well as objectual quantification, are formal and not material aspects of reality. Here already we begin to see the paradigmatic role of logical atomism, for in most other systems identity and difference, as well as objectual quantification, are also said to be formal in content, though propositions regarding that content are not also said to be either logically necessary or logically impossible.

Because our considerations will be restricted to quantifying over objects as concrete particulars and not, for example, over material properties and relations as well, the variant of logical atomism we shall discuss here is nominalistic. Several realist alternatives are sketched in order to highlight the significant theses and/or difficulties of nominalism, though it should be noted that not all forms of nominalism need agree with the special ontological theses of nominalist logical atomism.

Finally, it should also be noted that our concern in this chapter is with an adequate formal representation of the ontology of logical atomism and not with its theory of thought, meaning, or philosophy of language. We wish to leave open how these might or must be developed with respect to the system constructed here, especially

with regard to how they might or must pertain to the question of its logistic completeness." (pp. 244-247 of the reprint)

(1) The convention adopted here is to use scare-quotes when speaking of what connectives represent as 'properties' or 'relations'. This is done to mark a special philosophical use which is convenient in our informal discussion but which strictly speaking is ontologically misleading. A similar convention applies throughout when we refer to existence (being-the-case) and nonexistence (being-not-the-case) as material 'properties' of atomic situations.

(3) That is, an object's self-identity or nonidentity with any other object is invariant through all the possible worlds of a logical space containing that object. We must distinguish this ontological invariance from the varying semantical relation of denotation (*Bedeutung*) between an object and a (non-Tractarian) name or definite description of that object. The former must be accounted for within the formal ontology, the latter only within its applications.

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18. ———. 1975. "A Second Order Logic of Variable-Binding Operators." *Reports on Mathematical Logic* no. 5:13-18.

"It is well-known that Frege distinguished between first- and second-level concepts or functions. First-level concepts he associated with properties and relations between objects. These concepts Frege characterized as functions which assigned truth-values (the true or the false) to (n-tuples of) object(s) (1). An (n-tuple of) object(s) was said to fall under such a concept if it was assigned the true by that concept. In his *Begriffsschrift* these concepts were indicated by predicate variables. Second-level concepts or functions, on the other hand, correspond to variable binding operators, e.g., the universal quantifier or, as in Frege's later development of the *Begriffsschrift*, the course-of-values abstraction operator. The latter assigns to a monadic concept the class which is its extension while the former assigns a truth-value. Second-level concepts, i.e., second-level functions corresponding to variable-binding operators of the quantifier type, accordingly, can be associated with properties or relations between properties and relations of objects in a sense analogous to (but also different from) that in which first-level concepts are associated with properties or relations between objects. In Frege's terminology, while an (w-tuple of) object(s) is said to fall under a first-level concept, the latter is said to fall within, not under, a second-level concept if it is assigned the true by that concept.

Third-level concepts corresponding to quantifiers binding predicate variables were also introduced into the *Begriffsschrift*, but Frege seems to have had some doubts regarding their ontological or objective significance. Indeed, Frege's attitude toward third-level concepts seems in general to resemble the nominalists' attitude toward second-level concepts, viz., that they are merely formal or syncategorematic concepts which are immanent to the *Begriffsschrift* and correspond to nothing objective in reality.

The objectivity of first- and second-level concepts, however, was said by Frege to be "founded deep in the nature of things" (2). These concepts, in other words, have an objective and not merely a formal or syncategorematic content according to Frege. Accordingly, from the point of view of rendering one's ontological commitments explicit by means of appropriate quantifiers, this indicates that in a framework such as the *Begriffsschrift* we should allow not only for third-level quantifiers binding predicate-variables (having first-level concepts as their values), as Frege explicitly did allow, but also for fourth-level quantifiers binding second-level quantifier variables (having second-level concepts as their values), as Frege only implicitly allowed. This he did in effect by allowing free or schematic occurrences of second-level quantifier variables (as affixed to schematic individual variables)."

(...)

"Finally, we should perhaps point out that not all second-level concepts need be quantifier concepts. E.g., Frege himself took the "property" of being a property of the number 2 to be a second-level concept ([4], p. 75), and no doubt he intended there to be such a second-level concept corresponding to each and every object. In the present system we remain faithful to Frege's intentions. Indeed, by (CP-2), it is valid here that for each object x there exists a second-level concept within which fall all and only those first-level concepts under which x falls.

Our approach to the semantics of variable-binding formula operators differs in this regard from that of Mostowski [8], Thomason and Johnson [9], and Issel [5], [6], [7], all of whom, aside from restricting their considerations to first order languages (and, generally, to 1-ary 1-place quantifiers), interpret such operators as designating "quantities" of first-level concepts, i.e., they restrict their considerations to quantifier concepts.

The present system includes these sorts of second-level concepts but goes beyond them to include others as well. However, since the 'quantifier' terminology is simpler and more convenient than referring to variable-binding formula operators, we shall hereafter conflate the latter with the former and speak only of "quantifiers", though of course now quantifiers do not in all cases represent "quantities" (6)." (pp. 13-15)

(1) Frege apparently allowed only for binary relations. We extend his framework to include n -ary relations for arbitrary finite $n \geq 2$. In addition, we refer to all these relations as (n -ary) concepts. (Frege referred only to properties as concepts.)

(2) *Function and Concept*, p. 41 of [3].

(6) So-called branched quantifiers represent second-level concepts that are somewhat anomalous to quantifier concepts in general, i.e., to "quantities". It is well-known, however, that the semantic content of these quantifiers is representable in second order logic, and, accordingly, these concepts too are included among those represented in the system formulated below.

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19. ———. 1976. "A Note on the Definition of Identity in Quine's New Foundations." *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* no. 22:195-197.

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"A minimal second order modal logic of natural kinds is formulated. Concepts are distinguished from properties and relations in the conceptual-logic background of the logic through a distinction between free and bound predicate variables. Not all concepts (as indicated by free predicate variables) need have a property or relation corresponding to them (as values of bound predicate variables). Issues pertaining to

identity and existence as impredicative concepts are examined and an analysis of mass terms as nominalized predicates for kinds of stuff is proposed. The minimal logic is extendible through a *summum genus*, an *infima species* or a partition principle for natural kinds."

"A standard objection to quantified modal logic is that it breeds such reptiles of the mind as Aristotelian essentialism, "the doctrine that some of the attributes of a thing (quite independently of the language in which the thing is referred to, if at all) may be essential to the thing, and others accidental" ([5], p. 173f.). This objection has been criticized on one front by pointing out that none of the standard systems of quantified modal logic commit us to more than the meaningfulness of the non-trivial versions of the doctrine and that indeed we can, if we so choose, actually deny such versions in these systems (cf. [4]). A more heroic response, however, accepts these versions of the doctrine, at least when properly stated, and finds quantified modal logic the appropriate medium for its formulation. In what follows I shall attempt to formulate one such response, at least for the purpose of clarifying the general sort of logistic framework it presupposes if not also for exposing some of the more fascinating serpents that breed therein." (p. 202)

(...)

"Concluding Remarks. The above are only some of the more obvious principles that come to mind in the development and application of a logic of natural kinds. My objection to including them within the minimal system is based solely upon the rather strong sense of independence from the structure of thought (and therefore of "logic") which I assume natural kinds to have. Of course, in certain restricted contexts or for specialized applications these principles, and perhaps others as well, may be fully justified and used accordingly.

There are of course other developments and applications which I have not touched upon at all in this paper, e.g., the analysis of causal counterfactuals as based upon natural kinds or of a comparative similarity relation between individuals in terms of the natural kinds they share, etc. Our interests in these sorts of developments or applications should, it is hoped, vindicate at least to some extent the ontology of natural kinds as causal or nomological essences. In any case, such reptiles of the mind as these are taken to be by some philosophers seem hardly poisonous or deadly at all.

Finally, there is the sort of application suggested in section 4 for extending the logic of natural kinds to include nominalized predicates so as to provide a general analysis of the logic and ontology of mass terms. I have only hinted throughout this essay at how this richer framework might be developed, and though I do have some further suggestions which I have not gone into here, it is hoped that perhaps others will also take up the clarion call to defend this rather fascinating serpent of the mind." (. 220)

References

[4] Parsons, T. "Essentialism and Quantified Modal Logic." *Philosophical Review* 78 (1969): 35-52.

[5] Quine, W. "Three Grades of Modal Involvement." *Proc. XIth International Congress of Philosophy*, Brussels, 1953, vol. 14. Reprinted in *Ways of Paradox*. New York: Random House, 1966. Pages 156-174.

21. ———. 1977. "Sortals, Natural Kinds and Re-Identification." *Logique et Analyse* no. 80:439-474.

Reprinted in Lennart Aqvist and Franz Guenther (eds.), *Tense Logic*, Louvain: Nauwelaerts, 1978.

Abstract: "There are different ways in which we might investigate and represent the successive stages of the development of our common-sense and scientific conceptual frameworks. Jean Piaget's "fundamental hypothesis" regarding this development is that there is a parallelism between the progress made in the logical and rational organization of knowledge and the corresponding formative psychological processes" ([9], p.13). Piaget's approach has been a general inquiry into our formative psychological processes, a type of inquiry that requires us "to

take psychology seriously” (ibid., p.9). There is an alternative for philosophical logicians, however. For while it is not within our expertise to investigate formative psychological processes, we can nevertheless contribute to the study and representation of “the logical and rational organization of knowledge” through the construction of theories of logical form that are characteristic of at least some of the more important stages in the development of our common-sense and scientific frameworks. We adopt the methodology of such a construction in this paper where our primary concern will be the logical structure of our referential devices for quantifying, identifying and classifying things.

We will be concerned in particular with how this structure is to bear upon the problem of cross-world and cross-time re-identification."

"Investigations into the logical structure underlying ordinary language and our common-sense framework have tended to support the hypothesis that there are different stages of conceptual development and that while the structures elaborated at a later stage are in general not explicitly definable or reducible to those at the earlier they nevertheless presuppose them as conceptually prior bases for their own construction and elaboration— even when the conceptually prior structures are somehow eliminated or completely reconstructed at the later stages. This applies, moreover, not just to the conceptual structures underlying our common-sense framework but to those underlying the development of logic, mathematics and the different sciences as well.

Jean Piaget, for example, as a result of his investigations into genetic epistemology has found that our knowledge of logico-mathematical structures is obtained through a process of “constructive” or “reflective” abstraction that proceeds through a hierarchy of successive stages at which the structures acquired at a previous stage are reconstructed before they are integrated into the new structures elaborated at later stages (cp. [10], p.159). But, as Piaget has also shown, it is not just in logic and mathematics that cognitive activity develops through successive stages of progressive structuration; for the development of intelligence and knowledge in general, whether as represented in our common-sense or our scientific framework, proceeds in essentially the same way. Indeed, the construction of our scientific framework on the basis of our common-sense framework is itself a prime example not only of how conceptual structures acquired at a previous stage are completely reconstructed before they are integrated into those elaborated at a later stage but also of how the later structures, though built upon the earlier, cannot be reduced to or defined in terms of them (cf. Sellars [11]).

Now there are different ways in which we might investigate and represent the successive stages of the development of our commonsense conceptual framework. E.g., because of his “fundamental hypothesis” that there is a parallelism between the progress made in the logical and rational organization of knowledge and the corresponding formative psychological processes” ([9], p.13), Piaget’s approach has been a general inquiry into our formative psychological processes. The first principle of genetic epistemology, according to Piaget, is “to take psychology seriously” (ibid., p. 9).

There is an alternative for philosophical logicians, however. For while it is not within our expertise to investigate our formative psychological processes, we can nevertheless contribute to the study and representation of “the logical and rational organization of knowledge” through the construction of theories of logical form that are characteristic of at least some of the more important stages in the development of our common-sense and scientific frameworks.

One thing in particular that the construction of such a theory would help explain is the sense in which the operations and co-ordinations of concepts that characterize a given stage of conceptual involvement constitute a self-sufficient structured whole which purports to have limits beyond which there is nothing for thought. It would also help explain how the formalization of these operations and the clarification of their limits can be the basis for new and more elaborate operations whose structuration transcends those same limits and leads to a new stage of conceptual involvement.

It is this methodology that we shall adopt in what follows where our primary concern will be the logical structure of our referential devices for quantifying, identifying and classifying things. We shall particularly be concerned with how this structure is to bear upon the problem of cross-world and cross-time re-identification." (pp. 439-441)

References

[9] Piaget, Jean, *Genetic Epistemology*, translated. by E. Duckworth, Columbia University Press, New York, 1930.

[11] Sellars, Wilfrid, "Philosophy and the Scientific Image of Man," in *Science, Perception and Reality*, Routledge and Kegan Paul, London, 1963.

22. ———. 1978. "On the Logic of Nominalized Predicates and Its Philosophical Interpretations." *Erkenntnis* no. 13:339-369.
 Errata, *Erkenntnis*, 14, 103-104, pp. 103-104.
 "Predicate nominalizations are transformations of predicates and predicate phrases into nouns or noun phrases. Thus, e.g., 'pious' is transformed into 'piety', 'wise' into 'wisdom', 'triangular' into 'triangularity', and 'human' into 'humanity'. We call these types of derivative nouns abstract singular terms. Some relational predicates are also transformed into abstract singular terms: e.g., 'identity' for 'is identical with' and 'indiscernibility' for 'is indiscernible from'.
 There are other forms which predicate nominalizations take as well. E.g., the noun phrase 'the concept Horse', especially as used by Frege, amounts to a nominalization of the predicate 'horse', and others of a related sort are 'the property red' and 'the relation of being taller than'. These nominalizations have stylistic variations, e.g., 'redness' or 'red' simpliciter (when used as a singular term rather than as a predicate) and 'the taller-than relation' or simply 'being taller than'.
 There are no doubt a number of distinctions relevant to linguistics that should be drawn between these different types of nominalizations. We, however, shall not pursue them here but shall concern ourselves instead with the more formal question of a logic of nominalized predicates in the context of some of its philosophical interpretations. We shall assume in this regard that the occurrences of nominalized predicates in ordinary discourse for which the logic is designed are all singular terms in the modern sense, i.e., that they purport to have singular reference in the same sense in which proper names and (unreduced) definite descriptions are said to have such reference." (p. 339)
23. ———. 1979. "The Theory of Homogeneous Simple Types as a Second Order Logic." *Notre Dame Journal of Formal Logic* no. 3:505-524.
 "In its original form the theory of simple types, hereafter called ST, is a theory of predication and not, or at least not primarily, a theory of membership. With that original form in mind we construct in this paper a second order counterpart of ST which we call ST*. We briefly compare ST* with an alternative extension of second order logic, viz., the author's system T*(*) of [1], which was proposed as characterizing the original (and yet consistent!) logistic background of Russell's paradox of predication.
 In [2], the author showed the completeness of T**, plus an extensionality axiom (Ext*), relative to a Fregean interpretation of subject-position occurrences of predicates, viz., that such occurrences of predicates denote individuals correlated with the properties (or "classes") designated by predicate-position occurrences of the same predicates. It is observed here that when the semantical Fregean frames characterized satisfy ST*'s stratified comprehension principle instead of T**'s general comprehension principle, then the same Fregean interpretation yields a completeness theorem for monadic ST* + (Ext*) as well. It has been found convenient, on the other hand, to consider (monadic) ST as a theory of membership rather than a theory of predication when axioms of extensionality are included in its characterization. So considered, Quine proposed his system NF as a first order counterpart of ST, though of course, as is well-known, NF far exceeds ST in deductive powers. We show here per contra that while (monadic) ST* + (Ext*) is motivated in its construction along lines followed by Quine in the construction of

his first order counterpart NF, viz., the reduction of ST's metatheoretic feature of typical ambiguity to a stratified comprehension principle, our system, unlike NF, is equiconsistent with ST. This, along with the fact that the non-abstract individuals (or "urelements") of ST are retained unmodified in ST*, indicates that ST*, as a theory of predication, is to be preferred to NF, as a theory of membership, in the interpretation which each gives to ST's metatheoretic feature of typical ambiguity. We show in addition that if to (monadic) ST* +(Ext*) we add the assumption that whatever is a value of an individual variable is also (or, on the Fregean interpretation, is correlated with) a value of a (monadic) predicate variable, i.e., the assumption that every individual is a "class", then the resulting system is equiconsistent with NF. We refer to monadic ST* +(Ext*) as NFU* and show that it contains Jensen's system NFU as well." (pp. 505-506)

References

[1] Cocchiarella, N., "Whither Russell's paradox of predication?" in *Logic and Ontology*, M. K. Munitz, ed., New York University Press, New York (1973), pp. 133-158.

[2] Cocchiarella, N., "Fregean semantics for a realist ontology?" *Notre Dame Journal of Formal Logic*, vol. XV (1974), pp. 552-568.

24. ———. 1980. "Nominalism and Conceptualism as Predicative Second Order Theories of Predication." *Notre Dame Journal of Formal Logic* no. 21:481-500. "There appears to be a growing consensus, even if not unanimity, that standard predicative second-order logic is the appropriate logical medium for the representation of a nominalist theory of predication. We agree that this is indeed the case and formulate in this paper a model-theoretic approach which justifies that claim. (1) Because it is model-theoretic, our approach differs from the truth-value semantics approach of Leblanc and Weaver. (2) Amongst other reasons, we prefer our model-theoretic approach so as to accommodate those nominalists for whom the assumption that there are potentially as many names as there are individuals is not acceptable.

The models involved in our semantics, moreover, are precisely the same models as are already involved in standard first-order logic. Assignments of values (drawn from the domain of a given model) to the individual variables are extended, however, to what, relative to a given first-order language, we call *nominalistic assignments* to the n -place predicate variables (for each positive integer n) these assign first-order formulas (wffs) of the language in question, relative to the free occurrences of n distinct individual variables occurring in those wffs, to the H -place predicate variables. The satisfaction by such an assignment of a second-order wff in a model is then defined by a double recursion on the logical structure of the wff and on the number of nested predicate quantifiers occurring therein.

It is natural of course that a first-order wff, relative to n individual variables occurring therein as argument indicators, should be understood as representing an n -place predicate expression of the language in question; and in fact in an applied first-order theory based upon that language such a first order wff would constitute the definiens of a possible definition for an n -place predicate constant not already belonging to that language or occurring in that theory. Potentially, of course, there are infinitely many predicate constants that might be introduced into a first-order theory in this way; and it is just over such a potential infinity, and no more, that our predicate quantifiers, nominalistically interpreted, are understood to range when we turn to the predicative second-order counterpart of a given first-order theory. Finally, in order to better understand the implicit background of our nominalistic semantics, we include in a final section of this paper a brief comparison of nominalism, as represented by standard predicative second-order logic, with a closely related form of conceptualism, represented by a certain nonstandard predicative second-order logic formulated by the author in an earlier paper." (pp. 481-482)

(1) For the consensus view, see Parsons [9], For the dissenting view, at least in regard to the extension of predicative second-order logic to ramified type theory,

see Church [2].

It is possible of course that Church intends his demurral to apply only after predicates are ramified and allowed to occur as subjects of higher-order predicates. If so, then we believe that his demurral may have some merit (see Note 10).

(2) For reasons indicated in Note 10, we suspect that ramification may presuppose a linguistic capacity for introducing predicates that exceeds the proper limits of a nominalist theory of predication. Such a capacity does not exceed the limits of a closely related form of conceptualism (briefly discussed in Section 6) which may be represented by the nonstandard predicative second-order logic formulated in [3].

References

[2] Church, A., "Comparison of Russell's Resolution of the Semantical Antinomies with that of Tarski," *The Journal of Symbolic Logic*, vol. 41 (1976), pp. 747-760.

[3] Cocchiarella, N., "A new formulation of predicative second order logic," *Logique et Analyse*, vol. 17, no. 65-66 (1974), pp. 61-88.

[9] Parsons, C., "A plea for substitutional quantification," *The Journal of Philosophy*, vol. 68 (1971), pp. 231-237.

25. ———. 1980. "The Development of the Theory of Logical Types and the Notion of a Logical Subject in Russell's Early Philosophy." *Synthese* no. 45:71-115.

Reprinted as Chapter 1 in *Logical Studies in Early Analytic Philosophy*, pp. 19-63.

"The development of the theory of logical types in Russell's early philosophy proceeds along a difficult and rather involuted path; and even the final product, the theory as adumbrated in [*Principia Mathematica* = PM], remains unclear in its syntax and problematic in its semantics. Indeed, one might well be left with the impression that Russell himself, in the end, remained unsure of which parts of the different views he had held along the way are finally to be adopted.

In what follows, we shall attempt to describe and explain the development of Russell's early views, at least to the extent to which they are available in published form today, from the perspective of the development in those views of the notion of a logical subject. It is the development of this notion in Russell's early philosophy, we believe, that holds the key to many of the problems confronting Russell in the development of his theory of logical types and that led to the various, and sometimes conflicting, proposals that he made along the way.

It should be noted, however, that in referring to the development of the theory of logical types in Russell's early philosophy we have in mind only the views developed by Russell up to, but not subsequent to, the 1910—13 publication of the first edition of [PM]. The subsequent views developed by Russell from 1913—25, that is, between the first and second editions of [PM], and summarized to some extent in his introduction (and added appendices) to the second edition, constitute Russell's version of logical atomism. Except for some concluding remarks in the final section of this chapter, we delay our discussion of those views until chapter 5." (pp. 19-20 of the reprint)

26. ———. 1981. "Richard Montague and the Logical Analysis of Language." In *Contemporary Philosophy: A New Survey. Vol. 2: Philosophy of Language*, edited by Fløistad, Guttorm, 113-155. The Hague: Martinus Nijhoff.

"Richard Montague was an exceptionally gifted logician who made important contributions in every field of inquiry upon which he wrote. His professional career was not only marked with brilliance and insight but it has become a classic example of the changing and developing philosophical views of logicians in general, especially during the 1960s and 70s, in regard to the form and content of natural language. We shall, in what follows, attempt to characterize the general pattern of that development, at least to the extent that it is exemplified in the articles Montague wrote during the period in question.

The articles to which we shall especially direct our attention are: 'Pragmatics' [1]; 'Pragmatics and Intensional Logic' [2]; 'On the Nature of Certain Philosophical Entities' [3]; 'English as a Formal Language' [4]; 'Universal Grammar' [5]; and 'The Proper Treatment of Quantification in Ordinary English' [7].

Needless to say, but many of the ideas and insights developed in these papers Montague shared with other philosophers and logicians, some of whom were his own students at the times in question. Montague himself was meticulous in crediting others where credit was due, but for convenience we shall avoid duplicating such references here." (p. 113)

(...)

"Concluding Remarks

There are many other important features of Montague's grammar for English and of his translation of English by means of that grammar into intensional logic that we cannot go into here. The highly intensional nature of his semantics, for example, provides not only a more direct analysis of the opacity of intensional verbs but also a more direct analysis of the opacity of infinitive phrases as well. And then there is his treatment of relative clauses and of attributive adjectives, which we have not touched upon at all.

In closing then, it will no doubt have crossed the reader's mind that there may be some irony in the fact that Montague began his philosophical career as an extensionalist who took set theory as the proper theoretical framework for philosophy and as a formal-language philosopher who viewed the formalization of ordinary language as either impossible or extremely laborious, and in any case as certainly not philosophically rewarding. For the fact is that Montague has made important and philosophically innovative contributions toward a fully formalized syntax and semantics for natural language and that the semantics in question is most perspicuously described in terms of an intensional logic that transcends set theory and that in effect constitutes a new theoretical framework for philosophy. If this is not a revolution, it is at least a form of progress in the logical analysis of language." (p. 155)

References

[I] Works by Richard Montague

(The first 9 articles are reprinted in *Formal Philosophy, Selected Papers of Richard Montague*, edited and with an introduction by R.H. Thomason, Yale University Press, New Haven 1974. All page references here are to this volume. The dates listed are not the dates of publication but of when Montague first presented each paper to a philosophical audience.)

[1] "Pragmatics,"1964.

[2] "Pragmatics and Intensional Logic,"1967.

[3] "On the Nature of Certain Philosophical Entities,"1967.

[4] "English as a Formal Language,"1968.

[5] "Universal Grammar,"1969.

[6] "The Proper Treatment of Quantification in Ordinary English,"1970.

[7] "Syntactical Treatments of Modality,"1963.

[8] Montague and D. Kalish, "That,"1959.

[9] Montague and D. Kaplan, "A Paradox Regained,"1960.

[10] Montague and D. Kalish, *Logic: Techniques of Formal Reasoning*. New York: Harcourt, Brace and World, 1964.

27. ———. 1982. "Meinong Reconstructed versus Early Russell Reconstructed."

Journal of Philosophical Logic no. 11:183-214.

Reprinted as Chapter 3 in *Logical Studies in Early Analytic Philosophy*, pp. 119-151.

"Contemporary philosophy is in a rut, according to Terence Parsons in his recent book *Nonexistent Objects*, ([NO]), and it is one that stems from the (post-1905) work of Bertrand Russell. The main characteristic of this "Russellian rut" ([NO], 1) is strict adherence to the thesis that being, or being something, amounts to being something that exists—or equivalently that 'there is' is to be equated with 'there exists' ([NO], 6). This view is now so well entrenched, according to Parsons, that it is a main stay of what he also calls the orthodox tradition.

Now the orthodox view is in a rut, according to Parsons, "because it's a view in which most of us are so entrenched that it's hard to see over the edges" ([NO], 1).

Naturally, if we want “to look over the edge and see how things might be different” ([NO], 8), as any objective seeker of truth would, then “we need to encounter an actual theory about nonexistent objects” (ibid.). It is the construction and presentation of such a theory that is Parsons’s concern in *Nonexistent Objects*.
(...)

"Now we do not object to Parsons’s choice of Meinong’s theory here, nor for that matter to his elegant reconstruction and presentation of that theory. We do think, however, that a more balanced recognition of Russell’s overall view is called for and that perhaps the best way to make the Meinongian notion of a concrete object understandable to the orthodox tradition is to compare it with the general Russellian notion of a concrete individual, i.e., the Russellian notion of an individual that can exist but which might in fact not exist. Indeed, on the basis of the analysis and comparison we shall give here, it is our position that the Meinongian notion of a concrete object, at least as reconstructed by Parsons, is parasitic upon, though in a beneficent way, the Russellian notion of a concrete individual, existent or otherwise." (pp. 119-121)

References

[NO] Parsons, Terence, *Nonexistent Objects*, (New Haven and London: Yale University Press, 1980.)

28. ———. 1983. "Philosophical Perspectives on Quantification in Tense and Modal Logic." In *Handbook of Philosophical Logic. Vol. 2. Extensions of Classical Logic*, edited by Gabbay, Dov and Guentner, Franz, 309-353. Dordrecht: Reidel. Reprinted in Dov M. Gabbay and F. Guentner, *Handbook of Philosophical Logic, Second Edition*, Vol. 7, Dordrecht: Springer 2002, pp. 235-275. Contents: Introduction 235; 1. The Primary Semantics of Logical Necessity 236; 2. Logical Atomism and Quantified Modal Logic 237; 3. The Secondary Semantics of Metaphysical Necessity 240; 4. Proper Names as Rigid Designators 242; 5. Non-Contingent Identity and the Carnap-Barcan Formula 243; 6. Existence in the Primary and Secondary Semantics 245; 7. Metaphysical Necessity and Relational Model Structures 247; 8. Quantification with Respect to Individual Concepts 250; 9. Individual Concepts and the Elimination of *de re* Modalities 253; 10. Contingent Identity 258; 11. Quantifiers as Referential Concepts 258; 12. Singular Reference 259; 13. Conceptualism and Tense Logic 262; 14. The Problem of Reference to Past and Future Objects 266; 15. Time and Modality 268; Bibliography 274-275. "The trouble with modal logic, according to its critics, is quantification into modal contexts - i.e. *de re* modality. For on the basis of such quantification, it is claimed, essentialism ensues, and perhaps a bloated universe of *possibilia* as well. The essentialism is avoidable, these critics will agree, but only by turning to a Platonic realm of individual concepts whose existence is no less dubious or problematic than mere *possibilia*. Moreover, basing one's semantics on individual concepts, it is claimed, would in effect render all identity statements containing only proper names either necessarily true or necessarily false - i.e. there would then be no contingent identity statements containing only proper names. None of these claims is true quite as it stands, however; and in what follows we shall attempt to separate the chaff from the grain by examining the semantics of (first-order) quantified modal logic in the context of different philosophical theories. Beginning with the primary semantics of logical necessity and the philosophical context of logical atomism, for example, we will see that essentialism not only does not ensue but is actually rejected in that context by the validation of the modal thesis of anti-essentialism, and that in consequence all *de re* modalities are reducible to *de dicto* modalities.

(...)

Besides the Platonic view of intensionality, on the other hand, there is also a socio-biologically based conceptualist view according to which concepts are not independently existing Platonic forms but cognitive capacities or related structures of the human mind whose realization in thought is what informs a mental act with a predicable or referential nature. This view, it will be seen, provides an account in

which there can be contingent identity statements, but not such as to depend on the coincidence of individual concepts in the platonic sense. Such a conceptualist view will also provide a philosophical foundation for quantified tense logic and paradigmatic analyses thereby of metaphysical modalities in terms of time and causation. The problem of the objective significance of the secondary semantics for the analyzed modalities, in other words, is completely resolved on the basis of the nature of time, local or cosmic. The related problem of a possible ontological commitment to *possibilia*, moreover, is in that case only the problem of how conceptualism can account for direct references to past or future objects." (pp. 235-236)

29. ———. 1985. "Two Lambda-Extensions of the Theory of Homogeneous Simple Types as a Second Order Logic." *Notre Dame Journal of Formal Logic* no. 26:377-407.

Contents:

0. Introduction 377; 1. HST* revisited 379; 2. An improved axiom set for HST* 380; 3. The grammar of HST* with lambda-abstracts 383; 4. The system lambda-HST* 384; 5. The system HST*lambda 386; 6. An extensional Fregean semantics for nominalized predicates 390; 7. The relative consistency of HST*-lambda + (Ext*) to lambda-HST* + (Ext*) 393; 8. An intensional Fregean semantics for nominalized predicates 395; 9. Identity versus indiscernibility in HST*-lambda 402; Notes 406; References 407.

Abstract: "Two second order logics with lambda-abstracts are formulated as counterparts to the theory of homogeneous simple types. Predicates can be nominalized and occur as abstract singular terms in these logics so that self-predication is meaningful in general and, in certain special cases, even provable. Extensional and intensional Fregean semantics in which nominalized predicates are assigned individuals as concept-correlates are formulated and the extensional and intensional versions of these logics are shown to be complete with respect to their corresponding semantics. The logics are also shown to be consistent relative to weak Zermelo set theory."

"In the theory of simple logical types as originally conceived, it is meaningless for one predicate expression to occur in one of the subject or argument positions of another unless the latter is assigned a higher logical type than the former within the grammar of the object language; and therefore it is meaningless in particular for any predicate expression to apply to itself, i.e., to occur in one of its own subject positions. Russell's paradox of predication is thereby avoided, of course, but the price is high, for the resulting theory is not an accurate representation of the role of predicates in natural language where predicate expressions can apply not only to the nominalizations of other predicates but to their own nominalizations as well -- and without regard at all for the notion of a logical type. In the theory of logical types as a second-order logic, on the other hand, predicate expressions are typed within the grammar of the object language only in the way they are typed in standard second-order logic, i.e., only with respect to their degree or number of subject positions, and they are allowed otherwise to meaningfully occur in the subject or argument positions of other predicates, and of themselves as well, without regard to the notion of a logical type. Russell's paradox of predication can be avoided, it turns out, not by resorting to the notion of a logical type as a part of the grammar of the object language but rather only as a part of the metalinguistic description of the conditions under which properties and relations are to be posited by means of the grammar of the object language. The difference is crucial, needless to say, since it allows for a more accurate representation of the role of predicates and predication in natural language. The resulting theory is not, to be sure, a second-order logic in the "standard" sense used today (though it does contain the latter), but it is a second-order logic in the traditional or pre-type-theoretical sense in which quantifier expressions are allowed to reach into both subject and predicate positions without obliterating the logical and conceptually important distinctions between the two." (pp. 377-378)

30. ———. 1985. "Frege's Double Correlation Thesis and Quine's Set Theories NF and ML." *Journal of Philosophical Logic* no. 14:1-39.

Reprinted as Chapter 4 in *Logical Studies in Early Analytic Philosophy*, pp. 152-192.

"There are two fundamentally different notions of a class, which, following tradition, we might call the mathematical and the logical notions, respectively. The logical notion is essentially the notion of a class as the extension of a concept, and, following Frege, we shall assume that a class in this sense "simply has its being in the concept, not in the objects which belong to it" (Frege, [PW], 183)—regardless of whether or not concepts themselves differ, as Frege assumed, "only so far as their extensions are different" (ibid., 118). The mathematical notion of a class, on the other hand, is essentially the notion of a class as composed of its members, i.e., of a class that has its being in the objects that belong to it. This notion of a class, we claim, is none other than the iterative concept of set—or at least that is what it comes to upon analysis. Note that although what accounts for the being of a class under the one notion is not the same as what accounts for the being of a class under the other, nevertheless the axiom of extensionality applies equally to both notions. This means that the axiom of extensionality does not of itself account for the being of a class. (1)

Of course the logical notion of a class, especially as developed in Frege's form of logicism, is usually thought to be bankrupt as a result of Russell's paradox. This assessment, however, is erroneous. In particular, in "Frege, Russell, and Logicism: A Logical Reconstruction," ([FRL]), I have explained how Frege's view of classes in the logical sense can be reconstructed without paradox by modifying in either of two ways what I there referred to as Frege's double correlation thesis. The two systems that result from these modifications, it turns out, have certain structural similarities with Quine's two set theories NF and ML, especially when the latter are themselves modified so as to include urelements other than the empty set. This is significant because both NF and ML are commonly said to "lack a motivation" (cf. Boolos's "The Iterative Concept of Set" ([ICS]), 219). But that is because as theories of sets in the sense of classes that are composed of their members, which is really the only sense to which Quine is willing to commit himself, both NF and ML are incompatible with the iterative concept of set. As theories of classes in the logical sense, however, and in particular of the classes that Frege took to be the correlates of concepts, both NF and ML can be given a very natural motivation, especially when modified to include urelements. In what follows we shall defend this motivation by examining the structural similarities in question." (pp. 152-153 of the reprint)

(1) In an intensional language, the mathematical notion of a class might well assume a stronger axiom of extensionality, viz. one in which classes that are composed of their members are necessarily identical when they have the same members. Such an axiom would not in general hold for classes in the logical sense, since co-extensive concepts are not in general necessarily co-extensive. (It would of course hold for those classes in the logical sense that are the extensions of "rigid" concepts, i.e., concepts that have the same extension in every possible world.)

References

[ICS] G. Boolos, "The Iterative Conception of Set," *Journal of Philosophy* 68 (1971):215—31.

[PW] G. Frege, *Posthumous Writings*, eds. H. Hermes, F. Kambartel and F. Kaulbach; translated by P. Long and R. White (Oxford: Blackwell, 1979).

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